University of Portsmouth

Telecommuting Choice Modelling Using Fuzzy Rule Based Networks

by

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To my parents,

Faramarz and Mahnaz

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ABSTRACT

Telecommuting as an approach in transportation demand management has made the news a lot in recent years. Technology has enabled this growing trend, and more and more companies and families are taking advantages of it. Adopting telecommuting is a multidimensional decision making process that involves different aspects of life such as family, work and many more. Modelling telecommuting enables employers and employees to understand the main factors that influence on decision making about adopting telecommuting.

The role of subjective knowledge and linguistic variables cannot be ignored in human decision making process and Fuzzy Logic has proved to be a powerful tool for knowledge-based decision-making systems. Telecommuting as a multifaceted decision involves more on subjective knowledge rather that accurate numbers. Thus, fuzzy logic is applied for modelling telecommuting.

Moreover, the complex internal decision making process for adopting telecommuting reveals the role of various factors at different levels that influence on the outcome of the decision. Therefore, Fuzzy Rule Based Network, as a novel approach in modelling complex systems, is utilised. Using Fuzzy Network as a transparent approach, enables to understand the role of external inputs, intermediate variables and their interaction in modelling telecommuting.

According to choice theory and in order to find the maximum utilities of alternatives in telecommuting, the Fuzzy Network is tuned and optimised in terms of rules and membership function using Genetic Algorithm and Fuzzy c-mean clustering method. In addition, to reduce the size of Fuzzy Network, an input and branch selection method is proposed. Linguistic composition of the nodes in Fuzzy Network is also performed by an efficient method to reduce computational costs.

Results highlight the most important external and intermediate variables as well as decision rules in describing the suitability of telecommuting. Also, a Multinomial Logit model, as benchmark model, is developed to compare models performances which shows the superiority of the proposed method in transparency, efficiency and interpretability criteria.

The main contributions of this research can be highlighted in modelling the suitability of telecommuting using Fuzzy Rule Based Network, developing fuzzy utility model using Fuzzy Rule Based Network, tuning Fuzzy Rule Based Network using Genetic Algorithm, input and branch selection for Fuzzy Rule Based Network and finally proposing an efficient method for linguistic composition of Rule Based Network.

DECLARATION OF AUTHORSHIP

Whilst registered as a candidate for the above degree, I have not been registered for any other research award. The results and conclusions embodied in this thesis are the work of the named candidate and have not been submitted for any other academic award.

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Abbreviation

TDM: Transportation Demand Management (Page 10)

RBN: Rule Based Network (Page 12)

FN: Fuzzy Network (Page 13)

MF: Membership Function (Page 13)

MNL: Multinomial Logit Model (Page 13)

RUM: Random Utility Model (Page 13)

FUM: Fuzzy Utility Model (Page 14)

FID: Fuzzification- Inference - Defuzzification (Page 28)

SFS: Standard Fuzzy System (Page 28)

CFS: Chain Fuzzy System (Page 28)

HFS: Hierarchical Fuzzy System (Page 28)

SRBS: Single Rule Base System (Page 30)

MRBS: Multiple Rule Base Systems (Page 30)

Publications

Journal Papers:

[1]: Gegov, A., F. Arabikhan, and N. Petrov, *Linguistic Composition Based Modelling by Fuzzy Networks with Modular Rule Bases.* Fuzzy Sets and Systems, 2015. 269: p. 1-29.

[2]: Gegov, A., F. Arabikhan, and D. Sanders, *Rule base simplification in fuzzy systems by aggregation of inconsistent rules.* Journal of Intelligent and Fuzzy Systems, 2015. 3(3): p. 1331-1343.

[3]: Arabikhan F., M. Postorino, A. Dupont-Kieffer and A. Gegov, *Gender based analysis of zones of tolerance for transit service quality with emphasis on ITS*. Transportation Research Record Journal of the Transportation Research Board, 2016 (3). P. 73-80. ISSN: 0361-1981.

Conference Papers:

[4]: Arabikhan, F., M. Postorino, M. Kermanshah and A. Gegov. *Examining Employees' Preference toward Telecommuting with an Emphasis on Women Employees*. In *5th International Conference on Women Issues in Transportation*. 2014. Paris, France: Proceedings of the International Conference on Women Issues in Transportation: P. 563-572.

CHAPTER 1
TELECOMMUTING

1-1-Introduction

Cities are locations having a high level of accumulation and concentration of economic activities and are complex spatial structures that are supported by transport systems. The larger the city, the greater its complexity and potential for disruptions, particularly when this complexity is not effectively managed. The most important transport problems are often related to urban areas and take place when transport systems, for a variety of reasons, cannot satisfy the numerous requirements of urban mobility.

The most notable urban transport problems are: traffic congestion and parking difficulties, longer commuting, public transport inadequacy, difficulties for non-motorized transport, loss of public space, environmental impacts and energy consumption, accidents and safety, land consumption and freight distribution (Rodrigue, Comtois, & Slack, 2009).

Among mentioned problems, traffic and parking congestion has been weighted more by urban planners, traffic experts and city fellows. The solutions can be viewed by two different ways, first by increasing urban transport network capacity (supply augmentation) and the other is transport system management. In the former approach, by constructing roads, streets and highways, the traffic network capacity would be enhanced, however in long run, it is costly and will damage the environment. It will also increase the travel demand at the end. The latter, optimises the operation of current traffic network capacity in many different ways. The main two approaches in this area are:

1- *Transportation Supply Management (TSM):* Improve the traffic network efficiency by various strategies such as Intelligent Transportation System (ITS), Enhancing the quality of given services in transit using ITS [3], improvement of road geometry and many more.
2- *Transportation Demand Management (TDM):* The application of strategies and policies to reduce travel demand or redistribute this demand in space or in time. In this way, by decreasing the number of travels and vehicles, traffic congestion will lessen (Martin & Smith, 2008). TDM is not a panacea; it is the all-inclusive term given to this variety of

measures used to improve the efficiency of the existing transportation system (Gifford & Stalebrink, 2001).

Transport demand management has gained attention since the 1970s primarily as a result of significant increases in travel that have not been accompanied by increases in infrastructure capacity. TDM strategies target different aspects of travel such as trip generation, distribution, mode choice and route selection. In terms trip generation, the TDM objective is to eliminate trip partially and entirely (Gifford & Stalebrink, 2001).

Approaches in TDM in management and modification of commuting pattern, can be classified in four different groups:

- *Incentive-Based Methods*: various approaches such as implementation of pricing schemes to increase or decrease the use of certain transportation modes
- *Land use Management:* promoting smart growth and new urbanism through physical development
- *Improvement of Transportation Options*: such as adding bike lanes to busy arterials, providing secure bike storage facilities and so on.
- *Policies and Programs*: designed to target a certain change in travel behaviour such as telecommuting, high occupant vehicle priority and many more (Ferguson, 2007).

Living in technology era, people are equipped with various facilities to improve their lives' quality. Telecommuting is a product of this digital revolution and is one of the most famous approaches in programs and policies in TDM. It mainly introduces telecommunications as a substitution for travel (Ferguson, 2007).

1-2-Telecommuting Definition

Mokhtarian (1991a) has played an important role in introducing telecommuting. Telecommuting has been defined as the use of telecommunications technology to partially or completely replaces the commute to and from work to alleviate the massive twice-daily peak commuting loads imposed upon most major cities. Telecommuting has gained acceptance almost all around the world, both as a business response to internal business problems, and also as a transportation demand management (TDM) strategy.

Telecommuting is sometimes equated with teleworking which is simply the use of telecommunications-related technology to conduct work, but not all teleworking (e.g., teleconferencing) replaces a commute trip. In fact, teleworking may or may not replace travel at all. Telecommuting can also be considered as one type of remote work, which has been defined: work done by an individual while at a different location than the person(s) directly supervising and/or paying for it. Telecommuting, of course, is both Telework and remote work and can be categorized into two types: home-based and non-home-based (Mokhtarian, 1991a).

Moreover, Mokhtarian (Mokhtarian, 1991a) defined the criteria for determining whether a particular remote work is Telecommuting or not? Two indicators have been discussed: Tele (is the worker physically distant from the primary worksite (supervisor's location)?) and Commuting (is commute travel reduced or eliminated?). By these criteria, a remote work type would be considered telecommuting if it involves remote management and reduces commute travel.

The biggest obstacle of the widespread acceptance of telecommuting by employers has also been identified: "How will I know the employees are really working?" To the extent it can be shown that other, common forms of work have similar remote management requirements, the comfort level with telecommuting may be increased. Further, some management techniques for other kinds of work may be transferable to telecommuting (Mokhtarian, 1991a). From employer's side, the main barriers to use telecommuting has been identified: 1-concerns: management views about supervising remote work, security/privacy, and impact on profits. 2-Uncertainties: How tax laws may apply and 3- Application of workplace health and safety rules to home (General Accounting Office, 2001).

Nilles and his colleagues, the "fathers" of telecommuting, predicted that by the year 2000, 50% of US workers would be telecommuter (Jack, Carson, Grey, & Hanneman, 1976) , yet their prediction was wrong and a small fraction actually did until the target year. The number of employees reported to have worked from their home "on their primary job" in 2010 reported as 9.4 million (6.6 percent of the workforce), and up to now in US 10 percent of employees regularly do telecommuting from home (Bloom, Liang, Roberts, & Ying, 2013). Nilles and his colleagues indicated that many companies will implement telecommuting programs for their employees because such programs would allow them to increase productivity, reduce turnover, cut down on energy and real estate costs as well as many workers will enjoy increased flexibility and travel time and cost savings (Jack et al., 1976).

1-3-Impacts of Telecommuting

In this section the necessity of studying telecommuting is highlighted through reviewing its impact on different areas.

Early studies on telecommuting, indicate the following possible impacts:

- *Frequency*: decrease in work trip and may increase in non-commute trip
- *Time of day/day of week*: trips may be shifted to off-peak periods and different days of week to avoid congestion delays
- *Destination/length*: work trips may be made to a local centre rather than a down town office building; non-work trips may be made closer to home rather than closer to work
- *Mode*: closer trips to home may shift to non-motorized modes such as bicycle and walk, and if telecommuting helps flatten the peak for the use of transit modes, greater operational economies may result
- *Trip chaining patterns*: eliminating the work trip may break up efficient linked activity patterns, creating several one-stop trips instead of one multi-stop trip
- *Persons making trips*: household assignments may change, with the telecommuter perhaps taking on more trips because she/he is at home and available
- *Vehicle ownership*: in the medium term, the ability to telecommute may eliminate the need for a car or more likely a second car
- *Residential/job location*: in the long term, telecommuting may stimulate movement further from work to housing in more desirable and affordable outlying locations. Once the ability of telecommuting has been established, the worker may change jobs, moving to a more distant employer (Mokhtarian, 1991b).

Also, empirical research based on collected data in San Diego shows that the frequency of telecommuting varied from 9 to 58 percent (one day every two weeks to almost three days a week) with average of 23 percent (slightly more than one day a week per person). It was also mentioned that 3376 person–mile travel saved considering an average of 21.8 miles per telecommute occasion (average round trip of sample was 25.6 mile) (Mokhtarian, 1991b).

In another study, it was estimated that more than 40 percent of all urban trips were made by commuters and having that trip volume the air pollution would grow slightly more grey, the time spent commuting would grow longer, more energy would consume and the costs of commuting would increase. That study showed that the technological cost per employee to do telecommuting was from 300$ to 1000$ per annum and the annual commuting cost was from 650$ to 1600$. The impact of telecommuting was mentioned as: $10/week/employee lower, No free lunch, No parking was provided, higher productivity (estimated 13 percent) and better qualified applicants for available positions (Jack, Carlson, Gray, Hanneman, & University of Southern California, 1976).

The relative energy consumption advantage of telecommuting over commuting (the ratio of commuting energy consumption to telecommuting energy consumption) was 25.6 percent when the private automobiles was used and 12.1 percent for normally loaded mass transit was used. Also, numerous associated areas of impact was also determined including urban planning, resource allocation, environmental quality, social and industrial relationships, health care, mental health and productivity (Jack et al., 1976).

Kitamura and colleagues (1991) in their study used the data had been gathered in the late 1980s. They divided the sample in to two groups: one group did telecommuting and the other did not. The results showed that those employees who did telecommuting made far fewer trips than the others, an average of 1.94 versus 3.95 trip/day and household members did the same, though the difference was smaller, 3.08 versus 3.3 trips/day. Most of the trip reduction occurred during peak hours, telecommuters made 73 percent fewer morning-peak departure and 54 percent fewer afternoon departures. They also found out that the vehicle miles travelled (VMT) dropped from 53.7 miles to 13.2 miles on telecommuting days. However, telecommuters drove more miles per day, 56 versus 45 miles, which means that employees with longer commute tended to do telecommuting

In the other study, 40 home-based telecommuters and 58 employees who did not telecommute were compared. They found out that 27 percent of vehicle-trip and 77 percent of vehicle mile travelled (VMT) reduced among telecommuters. They also calculated the substantial emission reduction: 48 percent in total organic gases, 64 percent in carbon monoxide, 69 percent in NO_x and 78 percent in particular matter (Koenig, Henderson, & Mokhtarian, 1996).

California telecommuting program focused more on Centre-based telecommuting. That study shows that total vehicle miles travelled (VMT) was 53 percent lower on telecommuting days but the number of trips increased. They also mentioned that the emissions decreased by 15 percent (*ROG*), 21.5 percent (*CO*), 35 percent (NO_x) and 51.5 percent (*PM*) (Varma &

Mokhtarian, 1998). Another study was undertaken in Washington over 71 telecommuters, 8 centre-based and 63 Home-based, and 33 non-telecommuters. The results show that vehicle mile travelled (VMT) dropped by nearly 54 percent for telecommuters from offices and 66.5 percent for Home-based telecommuter. They also found out that NO_x and *PM* were reduced more than *ROG* and *CO* as they are highly correlated with VMT (Henderson & Mokhtarian, 1996).

Choo and colleagues (2003) through national aggregate data (1988-1998) estimated an economic time-series model of vehicle miles travelled (VMT) as a function of economic variables. Useful information on the relationship between telecommuting, travel behaviour and residential location decisions was gathered through national aggregate data in the United State of America (1988-1998). The average commute length was generally longer for telecommuters than non-telecommuters and the difference between them was increased: (i) Relocations made for a variety of reasons could lead to longer commutes thus prompting more telecommuting, (ii) increased availability of telecommuting might cause people to relocate farther from their jobs.

Walls, Nelson and Safirova (2005) analysed the data collected from e-commute program in 5 metropolitan areas of United States and looked at telecommuting frequency, mode choice and emission reduction. They calculated how much telecommuting would be needed to reach an annual volatile organic compounds (VOC) emission reduction target in each city. They also found out that by this amount of telecommuters, CO_2 would account for more than of total pollutants avoided. The reduction in NO_x and VOC would be fairly small with thanks to new generation of cleaner cars.

Apart from all mentioned transport impacts of telecommuting, like any other social phenomenon, it has advantages and disadvantages from individual, organisational and society point of views. The possible advantages and disadvantages of telecommuting has been studied (Wilton, Paez, & Scott, 2011) and presented in Table 1.1(Harpaz, 2002). Also, social factors of telecommuting has been studied and its impacts on improving quality of work-life has been indicated (Olorunfemi, 2013).

Moreover, The social influence of telecommuting has been studied and three mutually exclusive sources of social influence on the decision to telework has been identified as friends who telework, neighbours who telework, and colleagues at the workplace (Scott, Dam, Paez, & Wilton, 2012).

In addition, the impact of telecommuting on the city size and urban sprawl and also the influence of work force and those who adopt telecommuting in centralising and decentralising (shrinking and expanding) has been studied (Rhee, 2009).

The impact of telecommuting has also been studies on employee heath. In a longitudinal analysis they found out that Telecommuting health risks varied by telecommuting intensity. Non-telecommuters were at greater risk for obesity, alcohol abuse, physical inactivity, and tobacco use, and were at greater overall risk than at least one of the telecommuting groups. Employees who telecommuted 8 hours per month were significantly less likely than non-telecommuters to experience depression. There was no association between telecommuting and stress or nutrition (Henke et al., 2016).

The impact of telecommuting has also been studied from engagement and efficiency point of view. The availability of telecommuting in a longitudinal study over ten months was studied. The main finding indicate that individuals who worked in organizations that offer telecommuting were more engaged than those who worked in organizations that did not offer telecommuting. Furthermore, telecommuting availability was not only directly but also indirectly related to engagement via perceived supervisor goal support and goal progress. Engagement in general decreased over time. However, individuals who attained their personal work goals were able to maintain high levels of engagement.(Masuda, Holtschlag, & Nicklin, 2017)

Working at home could have many advantages for employees, employers and the communities. Telecommuting may offer fuller employment (by increasing the employability of circumstantially marginalized groups such as work at home parents and caregivers, the disabled, retirees, and people living in remote areas), reducing trips, traffic congestion and traffic accidents, relieving pressure on transportation infrastructure, lessening road repairs, reducing greenhouse gases and energy use. For companies, telecommuting expands the talent pool, increases productivity, reduces office expenses, carbon footprint and energy usage, turnover and absenteeism, improves employee morale and many more (Harpaz, 2002).

Table 1.1- Possible advantages and disadvantages of telecommuting for individual, organization and society
Source: Advantages and disadvantages of telecommuting for the individual, organization and society (Harpaz, 2002)

Level	Advantages	Disadvantages
Individual	• Autonomy/independence • Flexible working hours • Improvement in time management, professional flexibility	• Impaired feeling of belonging • Feeling of isolation • No separation between spheres of work and home

	• Saving is travel time and expenses • Flexibility in arranging supervision of family members, dependents	• Need for self-discipline • Lack of professional support • Impeded career advancement • Over-availability syndrome • Personality unsuitability • Legal issues
Organisation	• Increases productivity • Increased provision of human resources • Significant decrease in absence and tardiness levels • Savings in direct expenses • Increased motivation and satisfaction • Creation of a positive organisational image	• Application difficulties for organisations with centralized management • Investment in training and new supersession methods • Possible damage to commitment to, and identification with, the organisation • Changes in work methods • Costs involved in transition to telecommuting • Legal issues
Society	• Reduction of environmental damage • Decrease in traffic/congestion • Solution for special-needs populations • Savings in infrastructure and energy	• Creation of a detached society

The effectiveness of telecommuting has been studied from different perspectives through an extensive literature review (Allen, Golden, & Shockley, 2015). Telecommuting has been examined in six main categories: Work and family/nonwork (Job satisfaction, organizational commitment and identification, stress-related outcomes, performance, wages and career potential, withdrawal behaviours), Nature of work while telecommuting (flexibility/schedule control, autonomy, task interdependence), Interpersonal process (social and professional isolation, workplace relationships, knowledge sharing, innovation), Individual and family considerations, Organizational culture and support and finally Community and societal effects (traffic and emissions, business continuity, expanded work opportunities, societal ties). All advantages and disadvantages of telecommuting considering each category has been explained.

(Gloet & Blount, 2017)

According to reviewed studies and researches, telecommuting role in work environments is growing in light of its positive impacts on employees, employers and communities. That justifies more profound studies and modelling to make the most of this phenomenon.

1-4-Research Scope

These days, telecommuting is getting more popular among companies and employees in light of its positive impacts. The choice of adopting telecommuting does not exist for all the employees and the suitability of telecommuting should be assessed before offering such an option to an employee. The decision for adopting telecommuting incorporates concerns and perspectives of both the employer and the employee. Generally speaking, there are number of factors, motivations and constraints that influence the decision for choosing telecommuting. Working from home affects different aspects of employee’s life including family, work, commuting, social life and many more. Therefore, adopting telecommuting is a multidimensional decision and comprehensive models should be developed to describe and explain this complexity.

In describing and solving most of engineering problems varieties of deterministic/stochastic models have been developed to solve those complex problems. However, in solving real-life problems, linguistic information is usually encountered which is hard to quantify using classical mathematical techniques. Linguistic information is known as subjective knowledge and to be able to use this subjective knowledge into practical models usually different assumptions are made or in many cases, linguistic information is very often ignored.

Fuzzy logic has proven itself as a powerful tool in decision making using linguistic variables. As literatures show, it is believed that for making decision about adopting telecommuting, linguistic variables are utilised by people not precise values. Therefore, fuzzy logic is implemented in modelling suitability of telecommuting.

Since adopting telecommuting is a choice issue, a choice model will be develop which incorporate fuzzy logic. Also, model should be transparent or in other words, not only the influence of external inputs on output are important but also the interaction between intermediate variables and their role in describing the output should be examined. Thus, Rule Based Network will be adopted because of its capacity for transparent and white-box modelling.

In modelling telecommuting, large set of external inputs and intermediate variables should be taken into consideration which ends up with dimensionality issue for Rule Based Network. Therefore, a method should be considered to select external inputs as well as intermediate variables which means reducing the size of the model by removing non important variables and branches of the network. Optimised size of the model will be used for finding the maximum utility of alternatives which require the tuned Rule Based Network. In other words, a methodology should be set up to tune the RBN.

At the end, the results will be obtained from the developed model and it will be compared with existing Random Utility Models as benchmark models. Also different indicators will be utilised to assess the model performance.

1-5-Thesis structure

This thesis consist of 6 chapters namely, Telecommuting, Modelling telecommuting using random utility models, Modelling telecommuting using fuzzy systems, Statistical analysis of research data, Case study and Conclusion. In chapter 2, discrete choice modelling is discussed and existing researches for modelling telecommuting using random utility models are reviewed and possible gaps are highlighted. Chapter 3 focuses on the research approach for modelling the suitability of telecommuting using fuzzy systems and main contributions of this research are justified. In chapter 4, the data set which is used for developing proposed model in chapter 3 is statistically analysed. Chapter 5, as result chapter, the research methodology is validated using the data set described in chapter 4. In chapter 6 or conclusion, the main findings as well as the contribution of this research is indicated. At the end the possible future work and further research is highlighted.

CHAPTER 2
MODELLING TELECOMMUTING USING RANDOM UTILITY MODELS

2-1-Introduction

The purpose of this chapter is to create familiarity with current thinking and research on modelling telecommuting and justifies future research into a previously overlooked or understudied area. Modelling telecommuting has been studied from different perspectives. To understand how employees adopt telecommuting and to identify the most important factors that influence their decision, discrete choice models have been widely used by different scholars. Choice models are utilised when decision makers face with different alternative and have to pick the one which best serves their own interest. Adopting telecommuting is a choice problem which alternatives are frequency of telecommuting.

This chapter firstly focuses on disaggregate models and explains why that type of models should be considered for studying telecommuting. In the next sections, principles of choice and behavioural theory is discussed. Random Utility Model (RUM) is considered as a bench mark in choice modelling. So, RUM and its theoretical preliminaries as a black box model is described. Subsequently, choice model calibration and evaluation methods are illustrated.

In the main section of this chapter, previous studies and critical summary of published research literature relevant to modelling telecommuting, different conceptual frameworks for modelling telecommuting by pioneer scholars in this area as well as their main findings are illustrated. Also, main findings of some of the researches in modelling suitability of telecommuting are reviewed.

2-2-Dissaggregate Models

In recent years, the focus of studies in econometrics has been shifted from aggregate models which describe the market as a whole to disaggregate ones that consider individual decision-making units. Reasons for this shift have been studied in the literature and the main issues are:

- Market demand and supply are simply the aggregates of many individuals' action. Therefore, causal relations inherent in behaviour should be examined at the individual level.
- In the era of technology and data, access to household and individual data has become more convenient. Consequently, disaggregate models can be developed easier and individual behaviour could be examined more precisely.

In other words, disaggregate models are more often able to capture effects that cannot be obtained accurately in aggregate models (Train, 1986). Also, modelling of individual behaviour is either explicitly or implicitly at the core of all predictive models of aggregate behaviour (Ben-Akiva & Lerman, 1985).

Examining the suitability of telecommuting for employees is an important multidimensional decision that should be made at the individual level and based on various factors that influence the outcome of the decision. Thus, disaggregate model at individual level should be developed to be able to capture individual's attributes and attitudes.

2-3-Choice and Behaviour Theory

Suitability of telecommuting in terms of a number of the days that can be adopted by employees should be studied to maximise the benefit and also minimise its disadvantages. Based on job characteristics and requirements, personal and family issues employees are faced with a choice situation which alternatives are number of the days per week that they can work remotely from home.

Theories of individual choice behaviour have been discussed widely in the literature. Ben Akiva and Lerman (1985) has undertaken comprehensive studies in theories of behaviour in the context of disaggregate travel demand models and discrete choice analysis methods. They explain different approaches in theory of behaviour:

Descriptive: it makes assumptions how people behave but does not prescribe how they ought to behave

Abstract: it can be formalised in terms which are not specific to particular circumstances

Operational: It results in models with parameters and variables that can be measured and estimated.

In this study and among all existing approaches in theory of behaviour, operational method is implemented. Numerical models are developed and calibrated base on variables and their parameters.

Also, choice is an outcome of a sequential decision-making process which can be seen as the following steps:

1- Choice problem definition
2- Alternative generation
3- Evaluation of alternative evaluation
4- Choice
5- Implementation (Ben-Akiva & Lerman, 1985)

With regard to telecommuting case study, an employee is faced with a choice problem and has to think of number of the days per week is suitable for him or her to work from home or any other possible location apart from the primary workplace. In other words, alternatives are the number of the days per week which for studying the suitability of telecommuting are 0, 1, 2 and 3 or more days. In the next step of the decision process, the employee gathers the information about the attributes of every available alternative based on job characteristics, family relevant issues, manager's view and so many other parameters that influence his or her decision. All this information is then processed to arrive at a choice as a number of days which is suitable for an employee to adopt telecommuting. The final step in this decision-making process is obviously working from home (or any other workplace) for a chosen number of days.

Ben-Akiva and Lerman (1985) explain the four main elements that play the crucial role in choice theory:

1- **Decision maker:** The unit of decision making could be an individual person or a group. In the case of telecommuting, the decision maker is an employee who is the "actor" in more general sense. Employees face different choice situations and have different views towards the choice set.
2- **Alternative:** Choice set for the decision maker is a subset of a universal set. The choice set can be continuous and discontinuous. In the case of telecommuting, choice set is a discrete set which represents the possible number of days per week that an employee works from home.

3- **Attributes of alternatives:** The attractiveness of an alternative is evaluated in terms of a vector of attribute values. Attribute values are ordinal or cardinal. Also when alternatives are heterogenous (in telecommuting case study) and actors, decision makers, may have different choice sets, they evaluate different attributes and assign diverse values for the same attribute of the same alternative. So, general characteristics of each alternative are considered by its attributes.

4- **Decision rules:** It shows the internal mechanism used by the decision maker to process the information available and get to the unique choice. The main two categories for decision rules are noncompensatory and compensatory. In noncompensatory approaches like Satisfaction and Lexicographic rules (Tversky, 1972) a satisfaction criteria are formed and an alternative can be eliminated if it does not meet the criterion of at least one attribute. In compensatory decision rule like Dominance, an alternative is dominant to others if that is better for at least one attribute and not worse for all others. At best this rule can be used to remove the inferior alternatives from the choice set not to select the best one.

Utility is another compensatory decision rule which is widely used in choice models. In this approach, commensurability of attributes is considered. The attractiveness of each alternative is calculated by scalar which is called the utility of that alternative. This leads to defining a single objective function expressing the attraction of an alternative in terms of its attributes. The decision maker tries to **maximise** his/her utility through his/her choice. The assumption of a having utility index is based on the compensatory concept or trade off which is used by the decision-maker to compare different alternatives. In other words, a costly alternative may be chosen by individual if it compensates enough by offering better service.

The utility of each alternative (level of satisfaction) is obtained using utility function from two different methods: Ordinal or Cardinal. The former is a mathematical expression of a preference ranking of alternatives. Numerical values are assigned to the alternative which has no meaning except for the relationship of greater, less than or equal to. In the cardinal method, the utility of alternatives is measured in terms of a unit of utility (UTIL) which implies some uniqueness of its numerical values and therefore imposes more restriction than ordinal utility.

In choice theory, the decision maker is to behave rationally. In other words, individuals are consistent and transitive in the decision process and follow their own objectives in choosing alternatives. Generally speaking, decision makers are assumed to gather all the required information regarding with alternatives, recognise the constraints and limitations and make a consistent decision (Ben-Akiva & Lerman, 1985).

The concept of utility is widely used in choice models due to its numerical approach and comparability between different alternatives. In this study and for modelling suitability of telecommuting, the utility of each alternative is calculated using two methods (common approach and a novel method) which will be described in the following chapters.

2-3-1- Economic Consumer and Discrete Choice Theory

Microeconomic consumer theory introduces the basic approach to the mathematical theories of individual preferences. The main objective of this theory is to provide the means for the transformation of assumptions about desires into a demand function expressing the action of the consumer under given circumstances. Basically, it is assumed that consumers have the ability to compare all possible alternatives. So, there exists an ordinal utility function that expresses mathematically the consumer's preferences. The consumer selection of the most preferred alternative that satisfies the constraints can formulate mathematically as the maximisation of the utility of alternatives based on their attributes. Using different mathematical techniques, the demand functions are obtained which they maximise the utilities for alternatives (Layard & Walters, 1978).

Discrete choice theory (Ben-Akiva & Lerman, 1985) is based on consumer theory but with a discrete presentation of the set of alternatives. In choice theory, instead of deriving demand function, utility functions are directly worked with.

The most difficult assumption to make involves the form of the utility function. An additive utility function is most often assumed for convenience as follows:

$$U_n = \alpha X + \beta Y + \gamma Z$$

Where U is an ordinal utility function that maps the attributes values (X, Y and Z) to a utility scale for alternative n, also α, β and γ are the parameters that express the tastes if the decision maker.

Also for a rational individual decision maker, alternative 1 will be chosen if and only if:

$$U_1 > U_2 \text{ and } U_1 > U_3$$

When there is no difference (or smaller than some perceptual threshold level) between two utilities, the decision maker is indifferent between those two alternatives, and the choice is, therefore, indeterminate. The potential problem could be resolved by assuming that there is no possibility of a tie.

For finding the utilities of alternatives, numerical values should be assigned to given parameters in the utility function. This can be done by calibrating the model and functions having observations or revealed preferences. Different methods could be applied to estimate the coefficient such as maximum likelihood and least square.

2-3-2-Probabilistic Choice Theory

The first development of probabilistic choice theories was in the field of psychology. These theories were developed from the need to explain the experimental observation of inconsistent and non-transitive preferences. It has been observed that in choice experiments, decision makers have not selected the same alternative in the repetition of the same choice situations and also transitive preferences assumption have been violated. Therefore, the probabilistic choice theory was introduced to explain these inconsistencies

Ben Akiva and Lerman (1985) also mentions that the probabilistic approach can be utilised to capture the effects of unobserved variations among decision makers and unobserved attributes of alternatives. It has also been highlighted the pure random behaviour as well as errors due to incorrect perceptions of attributes and choices of suboptimal alternatives. Thus, probabilistic choice theories can be incorporated to address some of the weaknesses of consumer theory (Luce & Suppes, 1965).

Two approaches to the introduction of the probabilistic choice mechanism have been studied:

1- Constant Utility: Utilities are fixed and instead of selecting the alternative with the highest utility the decision maker is assumed to behave with choice probabilities defined by a probability distribution function over the alternatives that include the utilities as parameters (Luce & Suppes, 1965):
2- Random Utility: It is more in line with consumer theory. The decision maker is always assumed to select the alternative with the highest utility. However, the utilities are not known with certainty and are treated as random variables.
   Also, four sources of randomness has been identified:

- Unobserved attributes
- Unobserved taste variations
- Measurement errors and imperfect information
- Instrumental (or proxy) variables (Manski, 1977).

2-4-Random Utility Model

Ben Akiva and Lerman (1985)explain Random Utility Models. Random utility of an alternative can be expressed as a sum of observable (or systematic) and unobservable (disturbances) components:

$$U_{in} = V_{in} + \varepsilon_{in} \tag{1}$$

Where

U_{in}: Utility of alternative *i* for decision maker *n*

V_{in}: Observable component of utility for alternative *i* for decision maker *n*

ε_{in}: Unknown measurement error for alternative *i* for decision maker *n*

The choice probability of alternative *i* is equal to probability that the utility of alternative *i*, U_{in}, is greater than or equal to utilities of all other alternatives in the choice set:

$$P(i|C_n) = P\ [U_{in} \geq U_{jn}\, , all\ j \in C_n\ ] \tag{2}$$

Where C_n is the choice set for individual *n*.

From equations (1) and (2):

$$P(i|C_n) = P[V_{in} + \varepsilon_{in} \geq V_{jn} + \varepsilon_{jn},\ \forall j \in C_n\ ] \tag{3}$$

To drive a specific random utility model, it is required to make an assumption about the joint probability distribution of the full set of disturbances: $\{\varepsilon_{jn} \in C_n\}$.

To make the random utility model operational, it is important to stress that V_{in} and V_{jn} are functions and assumed to be deterministic. The other terms ε_{in} and ε_{jn} may also be functions, but they are random from the observational perspective.

For finding the utilities, reasonable functions should be considered for V that reflects any theory influences utility, though there is no imposed functional form on V. Also, use a function that has convenient computational properties that make it easy to estimate the unknowns (parameters).

$$
\begin{aligned}
V_{in} &= \beta X_{in} \\
V_{jn} &= \beta X_{jn}
\end{aligned} \tag{4}
$$

Where X_{in} and X_{jn} are the vectors of alternatives' attributes and β is vector of unknown parameters.

In addition, different assumption about distribution of ε and also differences of ε_{in} and ε_{jn} will lead to different choice models. In actual applications the disturbances will be composite of a great number of unobserved effects which contribute to the disturbance's distribution in some ways. Based on the different assumptions about the disturbance terms, three practical models are derived:

$\varepsilon_{in} - \varepsilon_{jn}$ uniform $\longrightarrow$ linear probability model

$\varepsilon_{in} - \varepsilon_{jn}$ normal $\longrightarrow$ binary probit model

$\varepsilon_{in} - \varepsilon_{jn}$ logistic $\longrightarrow$ binary logit model

Also worth noting that Luce (1959) and Tversky (1972) prove that if the choice axiom (if some alternatives are removed from the choice set, the relative choice probabilities from the reduced choice set are unchanged) holds and a utility measure is directly proportional to the choice probability, then there exists a "strict utility" model as follows:

$$
P(i|C_n) = \frac{U_{in}}{\sum_{j\in C_n} U_{jn}} \tag{5}
$$

Where

U_{in}: Utility of alternative *i* for decision maker *n*

$P(i|C_n)$: Probability of alternative *i* is chosen from choice set C_n

Random utility models (RUM) have been used in various fields of discrete choice modelling. As illustrated in previous section, the frequency of adopting telecommuting has always been the scope of modelling. Alternatives or frequency of adopting telecommuting are considered as discrete quantity and it is widely modelled by RUM.

2-4-1-Multinomial Choice

When choice set, C_n, can consist of more than two alternatives – like telecommuting where there are four alternatives (0,1,2 and 3 days or more) – the problem is called multinomial choice. In multinomial choice (Ben-Akiva & Lerman, 1985), it is not sufficient to simply specify the univariate distribution of the differences in error terms, hence it is required to characterize the complete joint distribution of all the disturbances. To reduce the complexity of the multinomial choice to a simpler one, $P_n(i)$ can express as:

$$U_{in} \geq U_{jn} \qquad \forall\, j \in C_n,\, j \neq i \ \text{ then}$$

$$U_{in} \geq \max U_{jn} \qquad \forall\, j \in C_n,\, j \neq i \tag{6}$$

Where $\max U_{jn}$ is the composite alternative out of all the elements in C_n other than i.

$$P_n(i) = P[V_{in} + \varepsilon_{in} \geq \max\ (V_{jn} + \varepsilon_{jn}),\ \forall j \in C_n, j \neq i\ ] \tag{7}$$

If U_{in} exceeds the utility if the composite alternative, then i is chosen, otherwise not.

The disturbance's distribution of composite alternative has to be derived to be able to utilise the above equation. By assuming the logistic distribution of error term, the multinomial logit can be obtained. This facet of the multinomial logit model leads to some of its most valuable properties and has made it the most widely used method for discrete choice analysis. The Multinomial Logit (MNL) model is expressed as:

$$P_n(i) = \frac{e^{V_{in}}}{\sum_{j \in C_n} e^{V_{jn}}} \tag{8}$$

Where $0 \leq P_n(i) \leq 1, \forall j \in C_n$ and $\sum_{i \in C_n} P_n(i) = 1$

2-4-2-Methods of estimation

To find the values of unknown parameters (β), choice models should be calibrated. There are number of methods which help to estimate the unknowns such as Maximum Likelihood and Least Square.

Maximum Likelihood: This method is used to estimate the values of unknown parameters for which the observed sample is most likely to have occurred. This method is computationally burdensome but conceptually is quite straight forward. Since the sample of

N observations is by assumption drawn at random from the whole population, the likelihood of the entire sample is the product of the likelihoods of the individual observation.

For multinomial logit:

$$y_{in} = \begin{cases} 0 & \text{If observation } n \text{ chose alternative } i \\ 1 & \text{Otherwise} \end{cases}$$

Likelihood function:

$$L^* = \prod_{n=1}^{N} \prod_{i \in C_n} P_n(i)^{y_{in}} \tag{9}$$

For linear in parameters multinomial logit:

$$P_n(i) = \frac{e^{\beta X_{in}}}{\sum_{j \in C_n} e^{\beta X_{jn}}} \tag{10}$$

Taking logarithm of L^* ($L = \log L^*$)

$$L = \sum_{n=1}^{N} \sum_{i \in C_n} y_{in} \left(\beta X_{in} - \ln \sum_{j \in C_n} e^{\beta X_{jn}} \right) \tag{11}$$

And finally by solving first order condition ($\frac{\partial L}{\partial \beta} = 0$), unknowns (β) will be obtained. Having unknown parameters, utilities can be calculated and probability of each alternative for each individual can be found out (Ben-Akiva & Lerman, 1985).

Least Square: This method is another way to estimate the unknown parameters, buy widely used in linear regressions. Least square estimators are those values of unknowns that minimise the sum of squared differences between the observed and expected values of observation.

By calibrating the choice models using Binary and Multinomial Logit/Probit models. Utility functions are calibrated and all the unknown parameters (coefficients) are identified (β). In case of being statistically significant, those variables are kept in the model and interpreted on how they influence on utility of each alternative.

2-4-3-Random Utility Model Evaluation

Number of approaches has been studies in literature for examining the goodness of fit of choice models such as multinomial logit model. In this study two famous indicators are considered:

1) $-2[L(0) - L(\beta)]$, where $L(0)$ is the value of the log likelihood function when all the parameters are zero and $L(\beta)$ which is the value of the log likelihood function at maximum. The whole terms is used to test the null hypothesis that all the parameters are zero. It is asymptotically distributed as chi-squared with K degrees of freedom.
2) ρ^2, which as informal goodness of fit index that measures the fraction of initial likelihood value explained by the model. It is defined as $\rho^2 = 1 - \frac{L(\beta)}{L(0)}$ which must lie between 0 and 1. ρ^2 is analogous to R^2 used in regression but it should be used with more caution (Ben-Akiva & Lerman, 1985).

2-5-Modelling telecommuting

Telecommuting has increased quickly in different countries (due to its various positive impacts as mentioned in previous section) though debates are still around this topic and many researches are being undertaken in this area from different points of view. It is crucial to develop reliable and realistic model to reflect which employees are more suitable for adopting telecommuting and also the frequency of doing that.

Mokhtarian and Salmon (1994) have undertaken a comprehensive study to understand the conceptual framework for adopting telecommuting. They studied and proposed the internal decision making process for adopting telecommuting. Two type of factors are important in making change and choice process: constraints (factor which prevents or hinders a change) or Facilitators (factor which allows change or make the change easier or more effective) and Drive (motivator: a factor which actually motivates a person to consider a change). The presence of facilitators increases the probability that telecommuting will be adopted, without that drive facilitators are assumed to have no effect on the adoption of telecommuting, the presence of constraints decreases the likelihood of adoption and if sufficiently strong, will preclude adoption.

Also, it has been discussed that internal decision making process for adopting telecommuting is initiated by some threshold level of dissatisfaction with one or more aspects of life. Such a search for solution is motivated by drives, and in case of dissatisfaction, it is the drives which activate a search for adjustment to reduce it. On the other hand, constraints are factors which inhibit the formation of preference. Drives and Constraints differ from two perspectives. First, drives are internal and some constraints are external which imposed by physical environment, other people or institutions. Second, while drives are long-term constructs, constraints may be temporary and be changed in very short time spans. Understandings the effect of constraints, in addition to the role of drives, is important for forecasting purposes. They also mentioned that constraints are those which, when they active, essentially preclude telecommuting from being chosen. A preference may express a long term priority that may not always be exercised. The main reason for mismatch between preference and action is assumed to be presence of constraints. The suitability model estimates the trade-offs between advantages and disadvantages of telecommuting that an individual must consider in forming his or her preference (Mokhtarian & Salmon, 1995; Mokhtarian & Salomon, 1997).

They used the data which were obtained from employees of the City of Diego. Drives and constraints were analysed based on factors in terms of advantages and disadvantages of telecommuting and also employee's general attitudes. They examined through explanatory variables in different categories such as work, family, independence and leisure, ideology, travel, awareness, managerial support, job suitability, technology availability, cost, social/professional interaction, household interaction, discipline/control and benefits of the commute. On the basis of mentioned variables, a binary Logit model was developed, and results described that by highlighting disability, stress, personal benefits as well as commute stress and time, preferences toward telecommuting would increase. On the other hand, workplace interaction, household interaction and commuting benefit would decrease employee's preferences on telecommuting (Mokhtarian & Salmon, 1995; Mokhtarian & Salomon, 1996b).

Along with the above research, Mokhtarian and Salmon (1994; 1996a; 1996b) focused more in drive and constraints in terms of telecommuting. They categorized drives into work, family, leisure, independence, ideology and travel and the constraints can be external (based in work environment) awareness, organization and job or internal (arising from psychological considerations) psycho-social variables.

Stanek and Mokhtarian (1995) concluded that Home-based telecommuting has been the most popular form of telecommuting to date, but it is unclear at this time which form of it will ultimately dominant. They supposed that the decision to telecommute was affected by individual drives and constraints. In their study, from 97 respondents, 61 were telecentre users, 15 did Home-based telecommuting, and 21 were non-telecommuters. Using attitudinal factor scores and analysis, as well as travel and sociodemographic variables, the preferences to telecommute from Home or appointed centres were modelled. They developed three Binary Logit Models and the results illustrated that preference for centre was associated with having personal benefits at the centre, a lack of autonomy at the regular workplace, a high amount of overtime, job suitability at the centre, older respondents, and the presence of children less than six years of age. In the second model, preference for home was associated with a lack of personal benefits at the centre, a strong work ethic at home, job frustration, less experience in the occupation, job suitability at home, the presence of children less than two years of age, and small household size. In the final model, preference for centre as opposed to home depended on personal benefits at the centre, work ethic at both home and the regular workplace, family care opportunities at home, and older respondents. Importantly, each model contained personal benefits from the centre, a measure of the independence drive, as a significant variable (Stanek & Mokhtarian, 1998).

Mahmassani et al (1993) also proposed another conceptual framework for adopting telecommuting. They showed in their exploratory research that adoption of telecommuting involves two principal categories of decision makers: the employee and the employer. Employee participation in telecommuting programs is in general considered to be voluntary; however, approval from supervisors is required. The employer's decision therefore plays a decisive role in the initiation of a telecommuting program. An exploratory analysis of executives' attitudes and stated preferences toward telecommuting were also presented. The results indicate that management issues such as employees' productivity, executives' abilities to supervise telecommuters, and data security remain barriers to the employer's adoption of telecommuting. The comparison between the stated preferences of executives and those of employees also shows that executives are more reluctant than employees to adopt telecommuting.

Bernardino and Ben-Akiva (1993) suggested a framework for home-based telecommuting adoption in which perspectives of the organization and the employees were incorporated. The proposed framework consists of three main components: environment, individual, and decision process. Later, Bernardino and Ben-Akiva (1996) proposed another comprehensive framework for the telecommuting adoption process consisting of three models: (a) employer design of the telecommuting program, (b) employer decision to offer the designed program to the employees, and (c) employee decision to adopt telecommuting.

Moreover, Suitability and propensity of adopting telecommuting have been studied in other studies. Random Utility Models have been used and different attributes of employees like age, number of children, gender, marital status, professional status and ideology towards telecommuting have been described as the main factors in determining the frequency of adopting telecommuting (Arabikhan, Kermanshah, Postorino, & Gegov, 2014; Drucker & Khattak, 2000; Mannering & Mokhtarian, 1995; Popuri & Bhat, 2003; Thériault, Villeneuve, Vandersmissen, & Rosiers, 2005; K. Walls, Douma, Loimer, Oslen, & Pansing, 2001; M. Walls, Safirova, & Jiang, 2007) and [4].

Mamdoohi et al. (2006) developed a model for suitability of telecommuting using the concept of abstract job. They introduce job as composed of tasks as found job titles not very revealing and the elements that a job comprises are considered to be effective in determining the level of telecommuting or it is suitable. An attempt is made to identify and group the basic tasks a job is composed of, pertaining to telecommuting suitability and to show the applicability of the approach by focusing in the identified job-tasks. Importance of PC, talking with colleague and clients, participating in meetings and missions out of office were the most important factors were identified for adopting telecommuting.

Based on reviewed literatures, adopting telecommuting is a multidimensional problem which is not confined to only few factors and involves different aspects of employee’s life such as work, family, commuting and so on. Among the existing conceptual frameworks, Mokhtarian and Salomon (1994) proposed the most comprehensive framework for modelling telecommuting and as discussed the internal decision making was illustrated. In that model, not only external inputs are important, but also existing intermediate variables were identified. As a case in point, different aspects of job as various external inputs have been highlighted but in decision making process job as an intermediate variable is considered more not those components individually. Mentioned internal decision making process and proposed framework is a transparent model which is taken into consideration in this study.

2-6-Conclusion

In this chapter the preliminaries of choice models for modelling telecommuting were discussed. Also, various studies that telecommuting has been modelled and studied using choice models were reviewed. All the discussed models are black-box, where inputs are fed to the model and outputs are obtained without knowing what is happening inside the model and what the process is. In the other words, no information is obtained for intermediate variables. This can be contradictory with what Mokhtarian and Salmon (1994) proposed for internal decision making process as the role of intermediate variables was highlighted in the established comprehensive conceptual framework.

As discussed, the current back box models cannot address the role of intermediate variables. So, the aim of this study is to develop a transparent model using the concept of choice models (multinomial logit) to obtain the probability of choices for decision makers. Moreover, as indicated, telecommuting is a multidimensional problem and decisions are more likely made by linguistic variables rather than numerical values. Therefore, in next chapter, fuzzy logic due to its power in describing decision maker’s behaviour is used to find the utilities of alternatives and probability of choices. Also, fuzzy systems are explained and Fuzzy Networks is introduced as an appropriate transparent method for modelling suitability of telecommuting.

CHAPTER 3
MODELLING TELECOMMUTING USING FUZZY RULE BASED NETWORKS

3-1-Introduction

This chapter focuses on the methodology and theoretical preliminaries of the research. In this study, the novel approach of Rule Based Network (RBN) is used for modelling the suitability of telecommuting. As adopting telecommuting is a choice problem and employees choose between their alternatives (frequency of doing telecommuting), the principles of choice and behaviour theory is followed which discussed in chapter 2.

The power of fuzzy logic is explained and justified how and why it can be incorporated with choice models. In the next step, the theoretical background of RBN is explained and it is shown why RBN is suitable to be used in fuzzy utility models and how its white-box nature helps to improve transparency and interpretability for modelling telecommuting.

In order to tune and calibrate RBN model, Generic Algorithm is proposed to be implemented. In addition, dimensionality as one of the main problems of RBN is discussed and backward selection method is suggested to tackle this problem in order to choose the most important external and intermediate variables and eventually reduce the size of model by cutting non-important branches of the network. To understand the performance of the proposed model, a Multinomial Logit model as a benchmark is also developed. Different performance indicators are introduced and models are compared using those criteria. Worth noting that the main contributions of this research are mostly discussed in Sections 3.6, 3.7 and 3.8.

3-2-Fuzzy Logic

Fuzzy logic was introduced by Zadeh (1965). He differentiated between logics and explained how classical logic is true or false and fuzzy logic is true to a degree. Fuzzy logic facilitates common sense reasoning with imprecise and vague propositions dealing with natural language and serves as a basis for decision analysis and control actions. Fuzzy logic focuses on linguistic variables in natural language and aims to provide foundations for approximate reasoning with imprecise propositions. It reflects both rightness and vagueness of natural language in common sense reasoning.

Decision making is a difficult process due to factors like incomplete and imprecise information, subjectively, linguistics, which tend to be presented in real-life situations to lesser or greater degree. These factors indicate that a decision-making process takes place in a fuzzy environment (Bojadziev & Bojadziev, 2007). In transportation field, different studies have been undertaken which fuzzy logic has been implemented to take advantages of its powerful attributes (Teodorović, 1999).

In terms of making decision for adopting telecommuting, it seems that fuzzy logic can be a promising tool for modelling. As explained in previous chapter, making decision for adopting telecommuting is a multidimensional case and employees use more linguistic and subjective approach rather than exact and precise numbers and values. Linguistic variable play an important role in making decision for telecommuting and the decision process can be understood better. In this study, the aim is to utilise fuzzy logic for modelling suitability of telecommuting.

In a study and in terms of modelling telecommuting using fuzzy logic, the inconsistency of if–then rules in adopting telecommuting is considered. Rules aggregation and simplification method has been proposed in this research and results show that the aggregated model with the consistent rule based approximates better than the original fuzzy system with the inconsistent rule based. Model accuracy criterion in terms of predicting the preference of doing telecommuting proves the proposed approach [2].

In modelling telecommuting, choice models have been used widely in the literature. In those models, utilities of alternatives have often been obtained by Logit or Probit models which are calibrated by maximum likelihood techniques – described in Section 2.3 – which works on a probabilistic approach. In other words, utility, which plays a crucial role in choosing an alternative, are modelled using probability. The question here is why probability should be used to find utilities of alternatives? Is there any other way to calculate utilities?

As Ross (2004) explains in his book, the basic statistical analysis relies on probability theory or stationary random process, while most experimental results contain both random (typically noise) and non-random processes.

Stationary random process has the following attributes:

- Sample and outcome space cannot change
- The frequency of occurrence, probability, of an event within that sample space is constant and cannot change from one experiment to other
- The outcome must be repeatable from experiment to experiment. The outcome of one experiment does not influence the outcome of a previous or future trail.

Stationary random process is those arise out of chance, where the chances represent frequencies of occurrence that can be measured. Problems like tossing coin and dice, picking the coloured ball out of a bag and similar trails are examples of the stationary random process. Obviously, humans do not make their daily decisions – mostly – on a random process. The uncertainty that decision makers face, are not categorised as random uncertainty. Humans often deal with uncertainty in daily life linguistically. Although this type of uncertainty can be captured by various classes of random stationary, probability, but solutions could not be reliable enough. Therefore, treatment of these forms of uncertainty using fuzzy set theory could be done with caution. Uncertainty in linguistic information or subjective knowledge can be explained well by fuzzy logic. Fuzzy sets provide a mathematical way to present vagueness in humanistic systems.

In the decision-making process, humans use subjective knowledge or linguistic information on a daily basis. In such problems, only objective knowledge (formulae and equations) or only subjective knowledge (linguistic knowledge) should not be used. Fuzzy logic is an extremely suitable concept with which to combine subjective and objective knowledge (Teodorović, 1999).

In adopting telecommuting, as a multidimensional decision, various factors such as family, job characteristics and responsibilities, commuting and so many other factors influence on the utility of each alternative (number of days per week). Employees evaluate their choices and all alternatives more on linguistic knowledge and such a situation can hardly be described by the stationary process. Author believes that decision makers deal more with subjective knowledge and consequently uncertainty in linguistic information is rather more challenging. Therefore, the fuzzy set theory could be justified to be implemented for obtaining utilities for an alternative for such a case study.

3-3-Fuzzy Systems and Networks

Fuzzy logic has proved itself as a powerful tool for dealing with uncertainty as an attribute of systemic complexity. In this context, fuzziness is quite suitable for reflecting non-probabilistic uncertainty such as imprecision, incompleteness and ambiguity (Fernandez-Caballero, 2009; Garibaldi & Ozen, 2007; Gil-Aluja, 2004). However, as far as structure is concerned, fuzzy logic is still unable to reflect adequately any interacting modules within a modelled process. This is due to the black-box nature of most fuzzy models that cannot take into account explicitly any interactions among sub-processes Fuzzy logic has also become more effective in dealing with dimensionality as a systemic complexity attribute by means of rule based reduction and compression. Dimensionality in rule based reduction is associated with the number of rules, which is an exponential function of the number of system inputs and the number of linguistic terms per input [1].

Fuzzy Rule Based Network (RBN) has been introduced recently as a theoretical concept. The nodes in these networks are modules of fuzzy rule bases and the connections between these modules are the outputs from some rule bases that are fed as inputs to other rule bases. The fuzzy network is characterised by a white-box nature whereby the inputs are mapped to the outputs by means of connections. RBN is a hybrid between Single Fuzzy System (SFS) and Chained/Hierarchical Fuzzy Systems (CFS/HFS). The structure of fuzzy RBN system is similar to the structure of CFS/HFS due to the explicit presentation of subsystems and the interactions among them. On the other hand, the operation of RBN resembles the operation of SFS as the multiple rule bases are simplified to a linguistically equivalent single rule base. This simplification is based on the linguistic composition approach. As a hybrid concept, RBN has the potential of combining the advantages of SFS and CFS/HFS (Gegov, 2010), [1].

Complexity is a versatile feature of existing systems that cannot be described by a single definition. In this context, complexity is usually associated with a number of attributes such as uncertainty, dimensionality and structure, which make the modelling of systems with these attributes more difficult. Therefore, the complexity of a given system can be accounted for by identifying the complexity related attributes that are to be found in this system (Gegov, 2007).

Fuzzy logic has proved itself as a powerful tool for dealing with uncertainty as an attribute of systemic complexity. In this context, fuzziness is quite suitable for reflecting non-

probabilistic uncertainty such as imprecision, incompleteness and ambiguity (Fernández-Caballero, 2009; Garibaldi & Ozen, 2007; Gil-Aluja, 2004).

More recently, fuzzy logic has also become more effective in dealing with dimensionality as a systemic complexity attribute by means of rule base reduction and compression. Dimensionality in rule base reduction is associated with the number of rules, which is an exponential function of the number of system inputs and the number of linguistic terms per input (Chen & Liu, 2005; Chen, Tong, & Liu, 2007; Wang & Lin, 2005; Wang & Luoh, 2004). In rule base compression, dimensionally is associated with the amount of on-line operations required during fuzzification, inference and defuzzification (Gegov, 2007).

However, as far as structure is concerned, fuzzy logic is still unable to reflect adequately any interacting modules within a modelled process. This is due to the black-box nature of most fuzzy models that cannot take into account explicitly any interactions among sub-processes (Kim & Cho, 2007;Xu & Shin, 2007; Zhang, Dang, & Li, 2009; Zhang, 2001). In this respect, the following paragraphs discuss some of the main approaches in fuzzy modelling and their ability to deal with structure as a systemic complexity attribute.

The most common type of fuzzy system is with a single rule base (Buckley, 2005; Piegat, 2001; Ross, 2004) . This type of system is referred to here as Standard Fuzzy System (SFS). The latter is characterised by a black-box nature whereby the inputs are mapped directly to the outputs without the consideration of any internal connections. The operation of SFS is based on a single Fuzzification-Inference-Defuzzification (FID) sequence and it is usually quite accurate for output modelling as it reflects the simultaneous influence of all inputs on the output. However, the efficiency and transparency of SFS deteriorate with the increase of the number of rules. Therefore, as the number of rules increases, it not only takes longer to simulate the model output but it is also less clear how this output is affected by the model inputs.

Another type of fuzzy system is with multiple rule bases (Duan & Chung, 2001; Lendek, Babuka, & Schutter, 2009; Mar & Lin, 2005; Yeh & Li, 2004). This type of system is often described by cascaded rule bases and it is usually referred to as Chained Fuzzy System (CFS) or Hierarchical Fuzzy System (HFS). Both CFS and HFS are characterised by a white-box nature whereby the inputs are mapped to the outputs by means of some internal variables in the form of connections. The operation of CFS and HFS is based on multiple FID sequences whereby each connection links the FID sequences for two adjacent rule bases.

CFS has an arbitrary structure in terms of subsystems and the connections among them (Bucolo, Fortuna, & Rosa, 2004; Hall, 2001; Igel & Temme, 2004). In this case, each subsystem represents an individual rule base whereas each interaction is represented by a connection linking a pair of adjacent rule bases. This connection is identical with an output from the first rule base and an input to the second rule base in the pair. CFS is usually used as a detailed presentation of SFS for the purpose of improving transparency by explicitly taking into account all subsystems and the interactions among them. Also, efficiency is improved because of the smaller number of inputs to the individual rule bases. However, accuracy may be lost due to the accumulation of errors as a result of the multiple FID sequences.

HFS is a special type of CFS that has a specific structure (Aja-FernÁndez & Alberola-LÓpez, 2008; Chen, Yang, Abraham, & Peng, 2007; Cheonga & Richard Laib, 2007; Joo & Lee, 2005; Lee, Chung, & Yu, 2003). Each subsystem in HFS has two inputs and one output. Some connections represent identical mappings, which may propagate across parts of the system. HFS is often used as an alternative presentation of SFS for the purpose of improving transparency by explicitly taking into account all subsystems and the interactions among them. Efficiency is also improved by the reduction of the overall number of rules, which is a linear function of the number of inputs to the subsystems and the number of linguistic terms per input. However, these improvements are often at the expense of accuracy due to the accumulation of errors as a result of the multiple FID sequences.

A third type of fuzzy system is with networked rule bases and it is referred to here as Networked Fuzzy System (NFS). This type of fuzzy system has been introduced recently as a theoretical concept in (Gegov, 2010) and it has been extended by more generic descriptions further in this work. NFS is characterised by a white-box nature whereby the inputs are mapped to the outputs by means of connections. Subsystems in NFS are represented by nodes and the interactions among subsystems are the connections among these nodes. NFS is a hybrid between SFS and CFS/HFS. On one hand, the structure of NFS is similar to the structure of CFS/HFS due to the explicit presentation of subsystems and the interactions among them. On the other hand, the operation of NFS resembles the operation of SFS as the multiple rule bases are simplified to a linguistically equivalent single rule base. This simplification is based on the linguistic composition approach that is described further in this work. As a hybrid concept, NFS has the potential of combining the advantages of SFS and CFS/HFS.

Properties of fuzzy systems such as accuracy, efficiency and transparency are directly related to attributes of systemic complexity such as uncertainty, dimensionality and structure. In this respect, uncertainty is an obstacle to (Joo & Sudkamp, 2009)accuracy as it is harder to build an accurate model from uncertain data (Landajo, Río, & Pérez, 2001; Lia, Shib, & Lia, 2002; Moreno-Velo, Baturone, Barriga, & Sánchez-Solano, 2007; Wan, Shang, Wang, & Sun, 2005). Furthermore, dimensionality represents an obstacle to efficiency because it is more difficult to reduce the amount of computations in a FID sequence for a large number of rules (Aluja, 2004; Joo & Sudkamp, 2009; Pal, Eluri, & Mandal, 2002; Xiong & Litz, 2002). Finally, structure is an obstacle to transparency as it is harder to understand the behaviour of a black-box model that does not reflect the interactions among subsystems (Fiordaliso, 2001; Gan, 2008; Guillaume, 2001; Kumar, Stoll, & Stoll, 2006).

In terms of fuzzy systems, Gegov et al. (Gegov, Petrov, Vatchova, & sanders, 2011) has undertaken a study to compare the performances Fuzzy Networks with other fuzzy systems. Three models have been developed: Single and Multiple rule based systems models (SRBS and MRBS), in addition to a rule based network model (RBN). The simulation results show that in terms of accuracy the SRBS model was the best, the MRBS model was slightly worse, whereas the MRBS was the worst of all. As far as transparency was concerned, the SRBS model is the worst, whereas the MRBS model and the RBN were better and equal to each other.

In the other study, Gegov and colleagues (2008) presented another application of fuzzy network into the procurement phase of the retail pricing process. The formation of product prices in the retail industry is a typical complex process. The most influential factor in the determination of product price is usually the maximum cost that the retailer pays to the manufacturer or the trader for the delivery of a product. In order to model this process and by considering maximum cost as an output and 3 inputs, they developed 3 different models: SRBS, MRBS and RBN. The results show that in terms of accuracy the SRBS and RBN models were more precise than MRBS model. From efficiency point of view, the MRBS model was the best, while the SRBS and the RBN models were worse. And finally, in terms of transparency the SRBS model was the worst, but the RBN and MRBS models were better.

Also, the linguistic composition of Fuzzy Network has been illustrated for transparent modelling for complex systems when intermediate variables exist. As a case study, a small model for telecommuting, as a complex system, has been developed. That study shows the ability of RBN in modelling complex systems where interaction between sub-processes and intermediate variables are important [1].

3-3-1- Fuzzy Network Theoretical Preliminaries

The theoretical preliminaries of Fuzzy Network has been discussed comprehensively in (Gegov, 2010) and [1] which is briefly explained in Sections 3-6-1 to 3-6-2-4 below:

A fuzzy system with *r* rules, *m* inputs $x_1 \ldots x_m$ taking linguistic terms from the input sets $\{A_{11},\ldots,A_{1r}\},\ldots,\{A_{m1},\ldots,A_{mr}\}$ and *n* outputs $y_1 \ldots y_n$ taking linguistic terms from the output sets $\{B_{11},\ldots,B_{1r}\},\ldots,\{B_{n1},\ldots,B_{nr}\}$ can be represented by the following rule base

$$\begin{array}{c} \textit{Rule 1: If } x_1 \textit{ is } A_{11} \textit{ and } \ldots \textit{ and } x_m \textit{ is } A_{m1}\textit{, then } y_1 \textit{ is } B_{11} \textit{ and } \ldots \textit{ and } y_n \textit{ is } B_{n1} \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \textit{Rule r: If } x_1 \textit{ is } A_{1r} \textit{ and } \ldots \textit{ and } x_m \textit{ is } A_{mr}\textit{, then } y_1 \textit{ is } B_{1r} \textit{ and } \ldots \textit{ and } y_n \textit{ is } B_{nr} \end{array} \tag{12}$$

A fuzzy network with *p.q* nodes $\{N_{11}\ldots N_{p1}\},\ldots,\{N_{1q}\ldots N_{pq}\}$, *p.q* node inputs $\{x_{11}\ldots x_{p1}\},\ldots,\{x_{1q}\ldots x_{pq}\}$ taking linguistic terms from any admissible input sets, *p.q* node outputs $\{y_{11}\ldots y_{p1}\},\ldots,\{y_{1q}\ldots y_{pq}\}$ taking linguistic terms from any admissible output sets, *p* horizontal levels and *q* vertical layers in the general grid structure for this network can be described by Equation (13)

$$\begin{array}{ll} & \textit{Layer 1}\ldots\ldots\ldots\ldots\textit{Layer q} \\ \textit{Level 1} & N_{11}(x_{11}, y_{11})\ldots\ldots\ldots N_{1q}(x_{1q}, y_{1q}) \\ & \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \textit{Level p} & N_{p1}(x_{p1}, y_{p1})\ldots\ldots\ldots N_{pq}(x_{pq}, y_{pq}) \end{array} \tag{13}$$

where the subscripts for the nodes specify their location in the grid structure and the subscripts for the associated inputs and outputs are identical with the ones for their nodes.

Each node in the grid structure from Equation (13) is a separate fuzzy system as the one described by Equation (12). The levels in this grid structure represent a spatial hierarchy of the nodes in terms of subordination in space and the layers represent a temporal hierarchy in terms of consecutiveness in time. For completeness, the fuzzy network described by Equation (13) has a node in each cell of the grid structure but in general, a grid structure may have empty cells.

The grid structure in Equation (13) does not give any information about the connections among the nodes in the fuzzy network. However, such information is contained by the sample connection structure in Equation (14) whereby the $p.(q-1)$ node connections $\{z_{11,12}\ldots z_{p1,p2}\},\ldots,\{z_{1q-1,1q}\ldots z_{pq-1,pq}\}$ take linguistic terms from the admissible sets for the associated node outputs and inputs

$$
\begin{array}{lc}
 & Layer\ 1 \ldots\ldots\ldots\ldots\ldots Layer\ q\text{-}1 \\
Level\ 1 & z_{11,12}=y_{11}=x_{12}\ldots\ldots\ldots z_{1q-1,1q}=y_{1q-1}=x_{1q} \\
 & \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
Level\ p & z_{p1,p2}=y_{p1}=x_{p2}\ldots\ldots\ldots z_{pq-1,pq}=y_{pq-1}=x_{pq}
\end{array}
\tag{14}
$$

where for each connection z the first subscript is identical with the subscript for its origin node and the second subscript is identical with the subscript for its destination node. Also, the first subscript for a particular connection z is identical with the subscript for the associated output y and the second subscript is identical with the subscript for the associated input x.

Like each node input and output from the general grid structure in Equation (13), each node connection from the sample connection structure in Equation (14) can be either of scalar or vector type. For simplicity, this interconnection structure describes only connections that are of feedforward type and among adjacent nodes in the same level but it can be easily extended for connections that are of feedback type or among non-adjacent nodes in different levels.

As a fuzzy network represents an extension of a fuzzy system, i.e. it can be viewed as a system of fuzzy systems or a network whose nodes are fuzzy systems, some of the general presentation techniques for fuzzy systems can be used also for fuzzy networks. However, other presentation techniques that are specific to fuzzy networks are required for the simplification of a fuzzy network to a linguistically equivalent fuzzy system. These techniques use compressed information about nodes in fuzzy networks and they are discussed further in this work.

3-3-2-Linguistic Composition Approach

The proposed linguistic composition approach uses Boolean matrices for the presentation of individual rule bases in fuzzy networks and operations on these matrices for manipulating the rule bases. A Boolean matrix compresses the information from a rule base that is represented by a node. In this case, the row and column labels of the Boolean matrix are all possible permutations of linguistic terms of the inputs and the outputs for this rule base. The elements of the Boolean matrix are either '0's or '1's whereby each '1' reflects a present rule. The Boolean matrix presentation of the rule base from Equation (12) is given by Equation (15).

$$\begin{array}{cccc} & B_{11}\ldots B_{n1} & \ldots & B_{1r}\ldots B_{nr} \\ A_{11}\ldots A_{m1} & 1 & \ldots & 0 \\ \ldots & & \ldots & \\ A_{1r}\ldots A_{mr} & 0 & \ldots & 1 \end{array} \quad (15)$$

The proposed approach uses also topological expressions for the overall presentation of fuzzy networks and the connections among the individual rule bases. Like grid and interconnection structures, topological expressions describe the location of nodes and the connections among them. In this case, the subscripts of each node specify its location in the network whereby the first subscript gives the level number and the second subscript gives the layer number. Besides this, topological expressions specify all inputs, outputs and connections for the nodes. The topological expression presentation of the fuzzy network from Equations (13)-(14) is given by Equation (16).

$$\begin{array}{c} \{[N_{11}]\ (x_{11} \mid z_{11,12}=y_{11}=x_{12})\ *\ \ldots\ *\ [N_{1q}]\ (z_{1q-1,1q}=y_{1q-1}=x_{1q} \mid y_{1q})\}\ + \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ +\ \{[N_{p1}]\ (x_{p1} \mid z_{p1,p2}=y_{p1}=x_{p2})\ *\ \ldots\ *\ [N_{pq}]\ (z_{pq-1,pq}=y_{pq-1}=x_{pq} \mid y_{pq})\} \end{array} \quad (16)$$

As shown in Equation (16), each node in a topological expression is placed within a pair of square brackets '[ ]'. The inputs and the outputs for each node are placed within a pair of simple brackets '( )' right after the node. In this case, the inputs are separated from the outputs by a vertical slash '|'. Nodes in the sequence are designated by the symbol '*' for horizontal relative location whereas nodes in parallel are designated by the symbol '+' for vertical relative location. Curly brackets '{ }' are used to specify the priority of linguistic composition operations in the fuzzy network, i.e. whether nodes with horizontal or vertical relative location have to be manipulated first.

Boolean matrices and topological expressions are very suitable for formal representation of fuzzy networks. While Boolean matrices describe fuzzy networks at a lower level of abstraction with respect to individual nodes, topological expressions describe these networks at a higher level of abstraction with respect to the whole network. In this context, Boolean matrices and topological expressions lend themselves easily to manipulation for the purpose of simplifying fuzzy networks to linguistically equivalent fuzzy systems using the linguistic composition approach. More details on this approach are presented below.

The linguistic composition approach is based mainly on the most common operations for the horizontal and vertical merging of nodes in fuzzy networks. These operations are binary in that can be applied to a pair of sequential or parallel nodes. Other less common operations such as output merging of nodes with common inputs are not considered in this work as they are not applicable to the case study. For simplicity, the operations of horizontal and vertical merging are illustrated for nodes with scalar inputs, outputs and connections but their extension to the vector case is straightforward. The operations make use of Boolean matrices at the node level and topological expressions at the network level.

3-3-2-1-Horizontal merging of rule bases

Horizontal merging can be applied to a pair of sequential nodes, i.e. nodes located in the same level of the fuzzy network. This operation merges the operand nodes from the pair into a single product node in the context of the linguistic composition approach. The operation can be applied when the output from the first operand node is fed forward as an input to the second operand node in the form of a connection. In this case, the product node has the same input as the one to the first operand node and the same output as the one from the second operand node whereas the connection does not appear in the product node.

The horizontal merging operation is identical with Boolean matrix multiplication. The latter is similar to conventional matrix multiplication whereby each arithmetic multiplication is replaced by a 'min' operation and each arithmetic addition is replaced by a 'max' operation. In this case, the row labels of the product matrix are the same as the row labels of the first operand matrix whereas the column labels of the product matrix are the same as the column labels of the second operand matrix.

Therefore, if the first operand node is the rule base from Equation (12) that is presented by the Boolean matrix from Equation (15) and the second operand node is the rule base in Equation (17) that is presented by the Boolean matrix in Equation (18),

$$\begin{array}{c} \textit{Rule 1: If } y_1 \textit{ is } B_{11} \textit{ and … and } y_n \textit{ is } B_{n1}\textit{, then } v_1 \textit{ is } C_{11} \textit{ and … and } v_g \textit{ is } C_{g1} \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ \textit{Rule r: If } y_1 \textit{ is } B_{1r} \textit{ and … and } y_n \textit{ is } B_{nr}\textit{, then } v_1 \textit{ is } C_{1r} \textit{ and … and } v_g \textit{ is } C_{gr} \end{array} \tag{17}$$

$$\begin{array}{cccc} & C_{11}\dots C_{g1} & \dots & C_{1r}\dots C_{gr} \\ B_{11}\dots B_{n1} & 1 & \dots & 0 \\ \dots & & \dots & \\ B_{1r}\dots B_{nr} & 0 & \dots & 1 \end{array} \tag{18}$$

the product node is the rule base in Equation (19) that is presented by the Boolean matrix in Equation (20)

$$\begin{array}{c} \textit{Rule 1: If } x_1 \textit{ is } A_{11} \textit{ and … and } x_m \textit{ is } A_{m1}\textit{, then } v_1 \textit{ is } C_{11} \textit{ and … and } v_g \textit{ is } C_{g1} \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ \textit{Rule r: If } x_1 \textit{ is } A_{1r} \textit{ and … and } x_m \textit{ is } A_{mr}\textit{, then } v_1 \textit{ is } C_{1r} \textit{ and … and } v_g \textit{ is } C_{gr} \end{array} \tag{19}$$

$$\begin{array}{cccc} & C_{11}\dots C_{g1} & \dots & C_{1r}\dots C_{gr} \\ A_{11}\dots A_{m1} & 1 & \dots & 0 \\ \dots & & \dots & \\ A_{1r}\dots A_{mr} & 0 & \dots & 1 \end{array} \tag{20}$$

In this case, the fuzzy system described by the rule base in Equation (17) is with r rules, n inputs $y_1 \dots y_n$ taking linguistic terms from the input sets $\{B_{11},\dots,B_{1r}\},\dots,\{B_{n1},\dots,B_{nr}\}$ and g outputs $v_1 \dots v_g$ taking linguistic terms from the output sets $\{C_{11},\dots,C_{1r}\},\dots,\{C_{g1},\dots,C_{gr}\}$. Similarly, the fuzzy system described by the rule base in Equation (8) is with r rules, m inputs $x_1 \dots x_m$ taking linguistic terms from the input sets $\{A_{11},\dots,A_{1r}\},\dots,\{A_{m1},\dots,A_{mr}\}$ and g outputs $v_1 \dots v_g$ taking linguistic terms from the output sets $\{C_{11},\dots,C_{1r}\},\dots,\{C_{g1},\dots,C_{gr}\}$. In general, the operand rule bases may have a different number of rules but the number of rules in the product rule base is always equal to the number of rules in the first operand rule base.

The horizontal merging operation above can be described by the block-scheme in Figure 3.1 and the topological expression in Equation (21)

$$[N_{11}]\ (x_1,\ldots,x_m \mid y_1,\ldots,y_n) * [N_{12}]\ (y_1,\ldots,y_n \mid v_1,\ldots,v_g) = [N_{11*12}]\ (x_1,\ldots,x_m \mid v_1,\ldots,v_g) \quad (21)$$

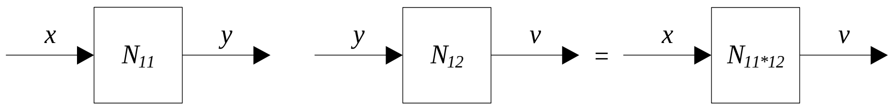


Figure 3.1: Horizontal merging of rule bases

where N_{11} and N_{12} are the two operand nodes from the fuzzy network and N_{11*12} is the product node for the fuzzy system. For simplicity, the notations used in Figure 3.1 are in a vector form where the vectors x, y and v are of dimension n, m and g, respectively.

Vertical merging can be applied to a pair of parallel nodes, i.e. nodes located in the same layer of the fuzzy network. This operation merges the operand nodes from the pair into a single product node. The operation can be applied when the outputs from the operand nodes are not fed as inputs to these nodes. In this case, the inputs to the product node represent the union of the inputs to the operand nodes whereas the outputs from the product node represent the union of the outputs from the operand nodes.

3-3-2-2-Vertical merging of rule bases

Vertical merging can be applied to a pair of parallel nodes, i.e. nodes located in the same layer of the fuzzy network. This operation merges the operand nodes from the pair into a single product node in the context of the linguistic composition approach. The operation can be applied when the inputs and the outputs of the two operand nodes are independent, i.e. there are no outputs that are connected to any inputs and vice versa. In this case, the inputs to the product node represent the union of the inputs to the operand nodes whereas the outputs from the product node represent the union of the outputs from the operand nodes.

The vertical merging operation is identical with Boolean matrix Kroneker product that represents an expansion of the first operand matrix along its rows and columns. In particular, the product matrix is obtained by expanding each non-zero element from the first operand matrix to a block that is the same as the second operand matrix and by expanding each zero elements from the first operand matrix to a zero block of the same dimension as the second operand matrix. In this case, the row labels of the product matrix are all possible permutations of row labels of the operand matrices whereas the column labels of the product matrix are all permutations of column labels of the operand matrices.

Therefore, if the first operand node is the rule base from Equation (12) that is presented by the Boolean matrix from Equation (15) and the second operand node is the rule base in Equation (22) that is presented by the Boolean matrix in Equation (23)

$$\begin{array}{c} \textit{Rule 1: If } v_1 \textit{ is } C_{11} \textit{ and } \ldots \textit{ and } v_g \textit{ is } C_{g1}, \textit{ then } w_1 \textit{ is } D_{11} \textit{ and } \ldots \textit{ and } w_h \textit{ is } D_{h1} \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \textit{Rule s: If } v_1 \textit{ is } C_{1s} \textit{ and } \ldots \textit{ and } v_g \textit{ is } C_{gs}, \textit{ then } w_1 \textit{ is } D_{1s} \textit{ and } \ldots \textit{ and } w_h \textit{ is } D_{hs} \end{array} \tag{22}$$

$$\begin{array}{cccc} & D_{11}\ldots D_{h1} & \ldots & D_{1s}\ldots D_{hs} \\ C_{11}\ldots C_{g1} & 1 & \ldots & 0 \\ \ldots & & \ldots & \\ C_{1s}\ldots C_{gs} & 0 & \ldots & 1 \end{array} \tag{23}$$

the product node is the rule base in Equation (24) that is presented by the Boolean matrix in Equation (25)

$$\begin{array}{c} \textit{Rule 1: If } x_1 \textit{ is } A_{11} \textit{ and } \ldots \textit{ and } x_m \textit{ is } A_{m1} \textit{ and } v_1 \textit{ is } C_{11} \textit{ and } \ldots \textit{ and } v_g \textit{ is } C_{g1}, \\ \textit{then } y_1 \textit{ is } B_{11} \textit{ and } \ldots \textit{ and } y_n \textit{ is } B_{n1} \textit{ and } w_1 \textit{ is } D_{11} \textit{ and } \ldots \textit{ and } w_h \textit{ is } D_{h1} \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \textit{Rule r. s: If } x_1 \textit{ is } A_{1r} \textit{ and } \ldots \textit{ and } x_m \textit{ is } A_{mr} \textit{ and } v_1 \textit{ is } C_{1s} \textit{ and } \ldots \textit{ and } v_g \textit{ is } C_{gs}, \\ \textit{then } y_1 \textit{ is } B_{1r} \textit{ and } \ldots \textit{ and } y_n \textit{ is } B_{nr} \textit{ and } w_1 \textit{ is } D_{1s} \textit{ and } \ldots \textit{ and } w_h \textit{ is } D_{hs} \end{array} \tag{24}$$

$$\begin{array}{cccc} & B_{11}\ldots B_{n1}D_{11}\ldots D_{h1} & \ldots & B_{1r}..B_{nr}D_{1s}..D_{hs} \\ A_{11}\ldots A_{m1}C_{11}\ldots C_{g1} & 1 & \ldots & 0 \\ \ldots & & \ldots & \\ A_{1r}\ldots A_{mr}C_{1s}\ldots C_{gs} & 0 & \ldots & 1 \end{array} \tag{25}$$

In this case, the fuzzy system described by the rule base in Equation (22) is with s rules, g inputs $v_1 \ldots v_g$ taking linguistic terms from the input sets $\{C_{11},\ldots,C_{1s}\},\ldots,\{C_{g1},\ldots,C_{gs}\}$ and h outputs $w_1 \ldots w_h$ taking linguistic terms from the output sets $\{D_{11},\ldots, D_{1s}\},\ldots,\{ D_{1h},\ldots, D_{hs}\}$. However, the fuzzy system described by the rule base in Equation (24) is with $r.s$ rules, $m+g$ inputs $x_1 \dots x_m$, $v_1 \dots v_g$ taking linguistic terms from the input sets $\{A_{11},\dots,A_{1r}\},\dots,\{A_{m1},\dots,A_{mr}\}$, $\{C_{11},\dots,C_{1s}\},\dots,\{C_{g1},\dots,C_{gs}\}$ and $n+h$ outputs $y_1 \dots y_g$, $w_1 \dots w_h$ taking linguistic terms from the output sets $\{B_{11},\dots,B_{1r}\},\dots,\{B_{n1},\dots,B_{nr}\}$, $\{D_{11},\dots, D_{1s}\},\dots,\{ D_{1h},\dots, D_{hs}\}$. The number of rules in the product rule base is equal to the product of the number of rules in the operand rule bases.

The vertical merging operation above can be described by the block-scheme in Figure 3.2 and the topological expression in Equation (26)

$$[N_{11}]\ (x_1,\dots,x_m \mid y_1,\dots,y_n) + [N_{21}]\ (v_1,\dots,v_g \mid w_1,\dots,w_h) = \\ [N_{11+21}]\ (x_1,\dots,x_m, v_1,\dots,v_g \mid y_1,\dots,y_n, w_1,\dots,w_h) \tag{26}$$

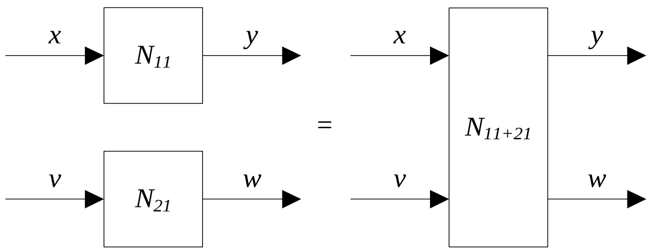


Figure 3.2: Vertical merging of rule bases

where N_{11} and N_{21} are the two operand nodes from the fuzzy network and N_{11+21} is the product node for the fuzzy system. For simplicity, the notations used in Figure 3.2 are in a vector form where the vectors x, y, v and w are of dimension n, m, g and h, respectively.

3-3-2-3-Associativity of rule base merging

The horizontal and vertical merging operations on nodes introduced above are quite basic in that they can be applied only to fairly simple fuzzy networks with a pair of nodes. However, a more complex fuzzy network may be with a large number of sequential and parallel nodes that have to be merged horizontally and vertically using the linguistic composition approach. This is possible due to the associativity property of the horizontal and vertical merging operations. These properties are proved below by theorems for scalar inputs, outputs and connections but the extension of the proofs to the vector case is straightforward.

The proofs presented below are based on the binary relational presentation of Boolean matrices. A binary relation compresses further the information from a Boolean matrix representation of a rule base. In this case, the pairs in the binary relation are the permutations of linguistic terms of the inputs and the outputs from the row and column labels for the

Boolean matrix. Therefore, each pair in the binary relation reflects a rule from the rule base. In this case, the Boolean matrices from Equations (15), (18), (21), (23) and (25) can be presented by the binary relations in Equations (27)-(31).

$$\{(A_{11}\ldots A_{m1},\ B_{11}\ldots B_{n1}),\ \ldots\ ,(A_{1r}\ldots A_{mr},\ B_{1r}\ldots B_{nr})\} \tag{27}$$

$$\{(B_{11}\ldots B_{n1},\ C_{11}\ldots C_{g1}),\ \ldots\ ,(B_{1r}\ldots B_{nr},\ C_{1r}\ldots C_{gr})\} \tag{28}$$

$$\{(A_{11}\ldots A_{m1},\ C_{11}\ldots C_{g1}),\ \ldots\ ,(A_{1r}\ldots A_{mr},\ C_{1r}\ldots C_{gr})\} \tag{29}$$

$$\{(C_{11}\ldots C_{g1},\ D_{11}\ldots D_{h1}),\ \ldots\ ,(C_{1s}\ldots C_{gs},\ D_{1s}\ldots D_{hs})\} \tag{30}$$

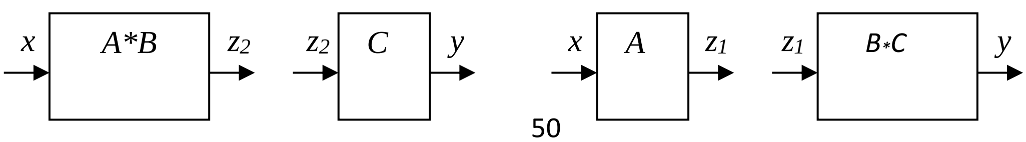
$$\{(A_{11}\ldots A_{m1}\, C_{11}\ldots C_{g1},\ B_{11}\ldots B_{n1}\, D_{11}\ldots D_{h1}),\ \ldots\ ,(A_{1r}\ldots A_{mr}\, C_{1s}\ldots C_{gs},\ B_{1r}\ldots B_{nr}\, D_{1s}\ldots D_{hs})\} \tag{31}$$

As binary relations are an alternative to Boolean matrices for representing nodes in fuzzy networks, they can also be used for horizontal and vertical merging operations on these nodes. In this case, horizontal merging is identical with standard relational composition whereas vertical merging is identical with a modified type of Cartesian product that is applied separately to the first and second elements from the pairs of the operand relations.

When the property of associativity is related to the operation of horizontal merging, the latter is applied to three sequential nodes for the purpose of merging them into a single node. In particular, this property allows the merging of three operand nodes *A*, *B* and *C* into a product node *A**B*C* to take place as a sequence of two binary merging operations that can be applied either from left to right or from right to left, as shown in Figure 3.3. The property can be applied when the output from the first node *A* is fed forward as an input to the second node *B* in the form of a connection and the output from the second node *B* is fed forward as an input to the third node *C* in the form of another connection. In this case, the product node *A**B*C* has the same input as the input to the first operand node *A* and the same output as the output from the third operand node *C* whereas the two connections do not appear in the product node.

Theorem 1: The operation of horizontal merging denoted by the symbol '*' is associative in accordance with Equation (32)

$$(A*B)*C = A*(B*C) = A*B*C \tag{32}$$



$*$ $=$ $*$

Figure 3.3: Associativity property of horizontal merging

whereby the horizontal merging of any three operand nodes *A*, *B* and *C* from left to right is equivalent to their horizontal merging from right to left.

When the property of associativity is related to the operation of vertical merging, the latter is applied to three parallel nodes for the purpose of merging them into a single node. In particular, this property allows the merging of three operand nodes *A*, *B* and *C* into a product node *A*+*B*+*C* to take place as a sequence of two binary merging operations that can be applied either from top to bottom or from bottom to top, as shown in Figure 3.4. The property can be applied when none of the outputs from any of the three nodes *A*, *B* and *C* are fed as any of the three inputs to these nodes. In this case, the input set to the product node *A*+*B*+*C* is the union of the inputs to the operand nodes *A*, *B* and *C* whereas the output set from the product node is the union of the outputs from the operand nodes.

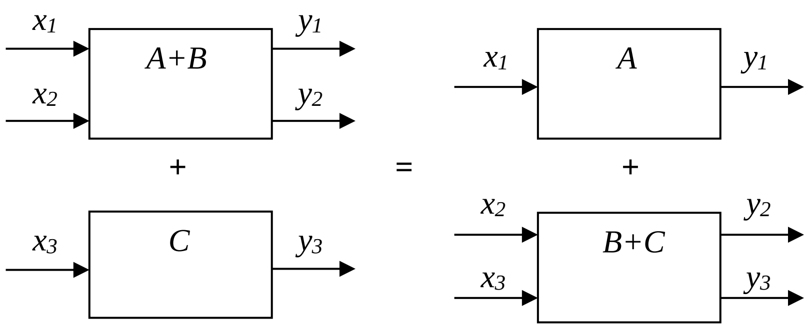


Figure 3.4: Associativity property of vertical merging

Proof of Theorem 1: The proof is based on the use of binary relations for representing the operand nodes *A*, *B* and *C*. In this case, the elements of the relational pairs are denoted by the letter *a* in *A*, the letters *a* and *c* in *B*, and the letter *c* in *C*, as shown in Equations (33)-(35). For clarity, all pairs in the middle relation *B* are assumed to be composable with pairs from the left relation *A* and the right relation *C*. This is why the first and the second element of each pair in *B* are denoted by *a* and *c*, respectively, and not by *b*.

$$A = \{(a_1^1, a_2^1),\dots,(a_1^p, a_2^p)\} \tag{33}$$

$$B = \{(a21, c11),\dots,(a21, c1q),\dots,(a2p, c11),\dots,(a2p, c1q)\} \tag{34}$$

$$C = \{(c11, c21),\dots,(c1q, c2q)\} \tag{35}$$

The first and the second element of any relational pair in A and C are denoted by the subscripts '*1*' and '*2*', respectively. However, the superscripts for the first and the second element of any relational pair in A and C are identical as they indicate the corresponding number for each pair. In particular, the relation A has p pairs and the relation C has q pairs. The subscripts for the first and the second element of any relational pair in B are '*2*' and '*1*', respectively. This is due to the requirement for left and right composability of B, i.e. the first element of each pair in B must be identical with a second element of a pair in A whereas the second element of each pair in B must be identical with a first element of a pair in C. In this case, the superscripts for the elements of the relational pairs in B don't have to be identical and therefore the relation B is assumed to have $p.q$ pairs.

The horizontal composition of the operand relations A and B gives the temporary relation $A*B$, as shown in Equation (36)

$$A*B = \{(a_1^1, c_1^1),\dots,(a_1^1, c_1^q),\dots,(a_1^p, c_1^1),\dots,(a_1^p, c_1^q)\} \tag{36}$$

Further on, the horizontal composition of the temporary relation $A*B$ and the operand relation C gives the product relation $(A*B)*C$, as shown in Equation (37)

$$(A*B)*C = \{(a_1^1, c_2^1),\dots,(a_1^1, c_2^q),\dots,(a_1^p, c_2^1),\dots,(a_1^p, c_2^q)\} \tag{37}$$

On the other hand, the horizontal composition of the operand relations B and C gives the temporary relation $B*C$, as shown in Equation (38)

$$B*C = \{(a_2^1, c_2^1),\dots,(a_2^1, c_2^q),\dots,(a_2^p, c_2^1),\dots,(a_2^p, c_2^q)\} \tag{38}$$

In this case, the horizontal composition of the operand relation A and the temporary relation $B*C$ gives the product relation $A*(B*C)$. As the latter is identical with the product relation $(A*B)*C$ from Equation (37), this implies Equation (32) and concludes the proof.

Theorem 2: The operation of vertical merging denoted by the symbol '+' is associative in accordance with Equation (39) whereby the vertical merging of any three operand nodes A, B and C from top to bottom is equivalent to their vertical merging from bottom to top.

$$(A+B)+C = A+(B+C) = A+B+C \tag{39}$$

Although Theorems 1-2 prove the associativity property only for fuzzy networks with three sequential and parallel nodes, respectively, this property can be trivially extended for fuzzy networks with an arbitrary number of nodes. Therefore, this property can be viewed in the context of the linguistic composition approach as the glue that makes the building blocks for simplification of a fuzzy network to a fuzzy system, i.e. the horizontal and merging operations on nodes, stick together. In this case, the generalisation of the associativity property for horizontal and vertical merging can be presented by Equations (40)-(41)

$$(((\dots((A*B)*C*)\dots*X)*Y)*Z) = (A*(B*(C*\dots*(X*(Y*Z))\dots))) = A*B*C*\dots*X*Y*Z \tag{40}$$

$$(((\dots((A+B)+C+)\dots+X)+Y)+Z) = (A+(B+(C+\dots+(X+(Y+Z))\dots))) = A+B+C+\dots+X+Y+Z \tag{41}$$

The associativity property of horizontal and merging operations from Theorems 1-2 provides the basis for the application of the linguistic composition approach to complex fuzzy networks with an arbitrary number of nodes. In particular, the nodes can be merged quite flexible, i.e. from left to right or right to left within the same level and from top to bottom or from bottom to top within the same layer. In this case, the resulting single equivalent system is the same irrespective of the order of application of the binary merging operations.

Proof of Theorem 2: The proof is based on the use of binary relations for representing the operand nodes *A*, *B* and *C*. In this case, the elements of the relational pairs are denoted by the letter *a* in *A*, the letter *b* in *B* and the letter *c* in *C*, as shown in Equations (42)-(44)

$$A = \{(a_1^1, a_2^1),\dots,(a_1^p, a_2^p)\} \tag{42}$$

$$B = \{(b_1^1, b_2^1),\dots,(b_1^q, b_2^q)\} \tag{43}$$

$$C = \{(c_1^1, c_2^1),\dots,(c_1^r, c_2^r)\} \tag{44}$$

The first and the second element of any relational pair in *A*, *B* and *C* are denoted by the subscripts '*1*' and '*2*', respectively. However, the superscripts for the first and the second element of any relational pair in *A*, *B* and *C* are identical as they indicate the corresponding number for each pair. In particular, the relation *A* has *p* pairs, the relation *B* has *q* pairs and the relation *C* has *r* pairs.

The vertical composition of the operand relations A and B gives the temporary relation $A+B$, as shown in Equation (45)

$$A+B = \{(a_1^1\, b_1^1,\, a_2^1\, b_2^1), \ldots, (a_1^1\, b_1^q,\, a_2^1\, b_2^q), \ldots, (a_1^p\, b_1^1,\, a_2^p\, b_2^1), \ldots, (a_1^p\, b_1^q,\, a_2^p\, b_2^q\,)\} \quad (45)$$

Further on, the vertical composition of the temporary relation $A+B$ and the operand relation C gives the product relation $(A+B)+C$, as shown in Equation (46)

$$\begin{aligned}(A+B)+C = \{&(a_1^1\, b_1^q\, c_1^1,\, a_2^1\, b_2^q\, c_2^1), \ldots, (a_1^1\, b_1^q\, c_1^r,\, a_2^1\, b_2^q\, c_2^r), \ldots,\\ &(a_1^p\, b_1^1\, c_1^1,\, a_2^p\, b_2^1\, c_2^1), \ldots, (a_1^p\, b_1^1\, c_1^r,\, a_2^p\, b_2^1\, c_2^r), \ldots,\\ &(a_1^p\, b_1^q\, c_1^1,\, a_2^p\, b_2^q\, c_2^1), \ldots, (a_1^p\, b_1^q\, c_1^r,\, a_2^p\, b_2^q\, c_2^r)\}\end{aligned} \quad (46)$$

On the other hand, the vertical composition of the operand relations B and C gives the temporary relation $B+C$, as shown in Equation (47)

$$B+C = \{(b_1^1\, c_1^1,\, b_2^1\, c_2^1), \ldots, (b_1^1\, c_1^r,\, b_2^1\, c_2^r), \ldots, (b_1^q\, c_1^1,\, b_2^q\, c_2^1), \ldots, (b_1^q\, c_1^r,\, b_2^q\, c_2^r)\} \quad (47)$$

In this case, the vertical composition of the operand relation A and the temporary relation $B+C$ gives the product relation $A+(B+C)$. As the latter is identical with the product relation $(A+B)+C$ from Equation (46), this implies Equation (39) and concludes the proof.

3-3-2-4-Application of rule base merging

The linguistic composition approach can be applied in the context of the three types of fuzzy systems discussed earlier – with single rule base, multiple rule bases and networked rule bases. This process consists of two stages whereby a multiple rule base system such as HFS is first converted into a networked fuzzy system such as FN and then the latter is composed of a single rule base system such as SFS. The theoretical validity of the above two-stage process is proved by means of topological expressions in Theorem 3 below whose proof is presented.

Theorem 3: A HFS with set of m inputs $\{x_1, x_2, \ldots, x_m\}$, a set of m-1 network nodes $\{N_{11}, N_{12}, \ldots, N_{1,m-1}\}$, a set of m-2 connections $\{z_1, z_2, \ldots, z_{m-2}\}$ and a single output y, as described by the block-scheme in Figure 3.5 and the topological expression in Equation (48)

$$[N_{11}]\ (x_1, x_2 \mid z_1) * [N_{12}]\ (z_1, x_3 \mid z_2) * \ldots * [N_{1,m-1}]\ (z_{m-2}, x_m \mid y) \quad (48)$$

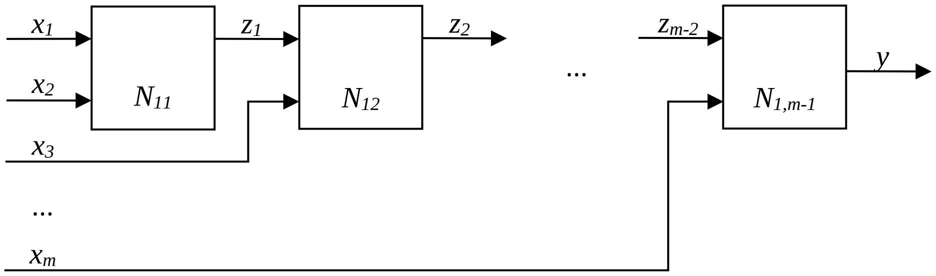


Figure 3.5: Hierarchical fuzzy system

can be represented as a SFS with the same set of *m* inputs, a single network node N, no connections and the same single output, as described by the block-scheme in Figure 3.6 and the topological expression in Equation (49)

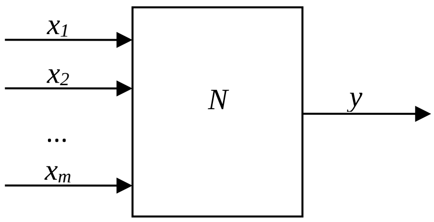


Figure 3.6: Standard fuzzy system

$$[*_{p=1}^{m-1} (N_{1p} + +_{q=p+1}^{m-1} I_{qp})] (x_1, x_2, \dots, x_m \mid y) \tag{49}$$

where $N = *_{p=1}^{m-1} (N_{1p} + +_{q=p+1}^{m-1} I_{qp})$.

Theorem 3 is applicable only to single-output systems but it can be extended trivially for multiple-output systems. In this case, the HFS would have a set of *n* outputs $\{y_1, y_2, \dots, y_n\}$ and it could be presented as a set of *n* independent systems. Therefore, the two-step process from the theorem above would be repeated for each independent system and its output.

Proof of Theorem 3: The HFS from Equation (36) can first be converted into a FN by representing all identity mappings propagating through any layers in the grid structure with the set of identity nodes $\{I_{21}\}, \dots ,\{I_{m-1,1}, I_{m-1,2}, \dots\}$. This FN can be described by the block-scheme in Figure 3.7 and the topological expression in Equation (50)

$$\begin{gathered} \{[N11]\ (x1, x2 \mid z1) + [I21]\ (x3 \mid x3) + \dots + [Im\text{-}1,1]\ (xm \mid xm)\} * \\ \{[N12]\ (z1, x3 \mid z2) + \dots + [Im\text{-}1,2]\ (xm \mid xm)\} * \\ \dots\dots\dots\dots\dots\dots\dots\dots * \\ [N1,m\text{-}1]\ (zm\text{-}2, xm \mid y) \end{gathered} \tag{50}$$

where each network node has two inputs and one output as opposed to each identity node that has one input and one output. In this case, the input to each identical node is identical to the output from the same node.

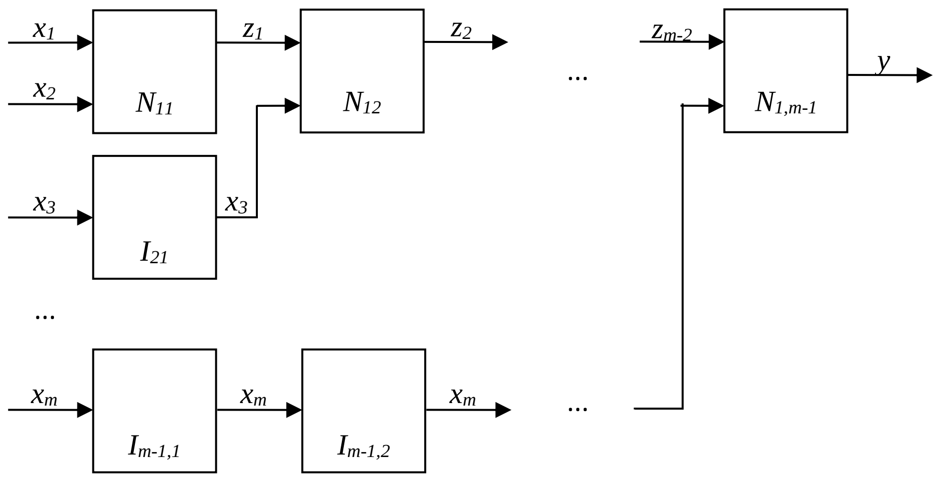


Figure 3.7: Fuzzy network

The FN can then be composed into a SFS by merging first vertically and then horizontally all network and identity nodes into a single network node $N = *_{p=1}^{m-1} (N_{1p} + +_{q=p+1}^{m-1} I_{qp})$. In this case, the SFS is like a single node FN with the same set of m inputs $\{x_1, x_2, \ldots, x_m\}$ and the same single output y as the HFS. This SFS can be described by the topological expression from Equation (49) that uses prefix notation for the horizontal merging operation and a mixture of infix/prefix notation for the vertical merging operation. This concludes the proof.

3-3-2-5-Efficient method for rule base merging

For large networks with several levels and layer, horizontal and vertical merging will end up with large Boolean matrices which is very computationally expensive. That happens as horizontal merging is identical with Boolean matrix multiplication (as explained in Section 3-6-2-1) and in vertical merging the product matrix is the permutation and union of the rows and columns of operand matrices (see 3-6-2-2).

Inefficiency of rule base merging using full matrices is illustrated in a simple example. In this example, Boolean matrix A with two inputs (X_1 and X_2), one output (Y_1) and two linguistic variables (1 and 2), is vertically merged with matrix B with one input (X_3), one output (Y_2) , two linguistic variables (1 and 2) and finally vertically merged A+B matrix is merged horizontally with matrix C with two inputs, one output (Y_3) and two linguistic variables (1 and 2) shown in Figure 3.8.

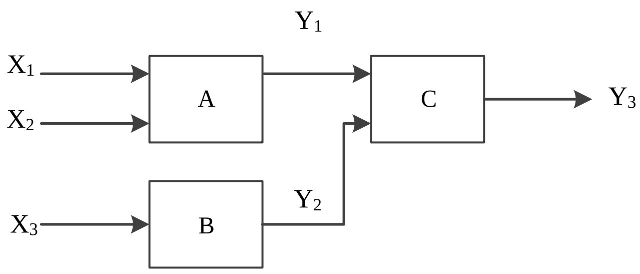


Figure 3.8: Horizontal and vertical merging – Example

Given matrix A= $\begin{array}{cc} & Y_1 \longrightarrow \\ X_1, X_2 \downarrow & \begin{array}{c}11\\12\\21\\22\end{array}\begin{bmatrix}0 & 1\\0 & 0\\1 & 0\\0 & 0\end{bmatrix}\end{array}$ and B = $\begin{array}{cc} & Y_3 \longrightarrow \\ X_3 \downarrow & \begin{array}{c}1\\2\end{array}\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\end{array}$ and modular base C is equal with

C= $\begin{array}{cc} & Y_3 \longrightarrow \\ Y_1, Y_2 \downarrow & \begin{array}{c}11\\12\\21\\22\end{array}\begin{bmatrix}0 & 0\\1 & 0\\0 & 0\\0 & 1\end{bmatrix}\end{array}$ where columns 1 and 2 in all matrices show linguistic variable 1 and 2.

First, matrices A and B should be merged vertically:

$$A + B = \begin{array}{cc} & Y_1, Y_2 \longrightarrow \\ X_1, X_2, X3 \downarrow & \begin{array}{c}111\\112\\121\\122\\211\\212\\221\\222\end{array}\begin{bmatrix}0 & 0 & 1 & 0\\0 & 0 & 0 & 1\\0 & 0 & 0 & 0\\0 & 0 & 0 & 0\\1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 0 & 0\\0 & 0 & 0 & 0\end{bmatrix}\end{array}$$

where columns 1 till 4 show all possible permutations of 1 and 2 which are 11, 12, 21 and 22 respectively. Consequently the equivalent single rule base is obtained by merging horizontally matrices A+B and C:

$$(A + B)*C = \begin{array}{cc} & Y_3 \longrightarrow \\ X_1, X_2, X3 \downarrow & \\ \begin{matrix}111\\112\\121\\122\\211\\212\\221\\222\end{matrix} & \begin{bmatrix}0 & 0\\0 & 1\\0 & 0\\0 & 0\\0 & 0\\1 & 0\\0 & 0\\0 & 0\end{bmatrix}\end{array}$$

where columns in above matrix are 1 and 2 respectively which show the linguistic terms. Above is the linguistic equivalent rule base.

As can be seen from above very small example, there are several zero-rows in matrices which are only additional load to computational operations. In bigger size and data-driven rules problems where rules are extracted from data (not expert knowledge), several zero rows can be seen in matrices. Having large matrices with several zero rows in large networks cause very expensive computations to merge the subsystems and generate massive equivalent rule base with only few non-zero rows.

An efficient method is proposed here and will be used in this research. The concept of this method is to remove the zeros rows in matrices and also label the permutations of rules' antecedents in each modular bases before vertical merging. Also in vertical merging, those permutations are made that their rules' consequents exist in the rules antecedents of the modular bases in the next layer which they are fed to. The reason behind that refer to horizontal merging. In horizontal merging those zero rows in the next layer matrix generate zero rows which are redundant. This procedure can be mentioned in the following steps using the above example:

a) Removing zero rows:

Matrix A will be A= $\begin{matrix}11\\21\end{matrix}\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$, B= $\begin{matrix}1\\2\end{matrix}\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$ and Matrix C= $\begin{matrix}12\\22\end{matrix}\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

b) Labelling permutations in matrix A, 11 $\longrightarrow$ 1 and 21 $\longrightarrow$ 2:

$$A= \begin{matrix}1\\2\end{matrix}\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}, B= \begin{matrix}1\\2\end{matrix}\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} \text{ and } C= \begin{matrix}12\\22\end{matrix}\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$

c) Vertical merging to produce non-zero rows in horizontal merging (those permutations of rule antecedents in matrices A and B that generate consequents 12 and 22 as antecedents of matrix C):

$$A + B= \begin{array}{c} 12 \\ 22 \end{array} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

where columns 1 and 2 represent 12 and 22 as consequent.

d) Horizontal merging:

$$(A + B)*C= \begin{array}{c} 12 \\ 22 \end{array} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

e) Replacing the labels: 1 $\longrightarrow$ 11 and 2 $\longrightarrow$ 21

$$(A + B)*C= \begin{array}{cc} & Y_3 \longrightarrow \\ X_1, X_2, X3 \downarrow & \begin{array}{c} 112 \\ 212 \end{array} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \end{array}$$

As can be seen above, the equivalent linguistic rule is identical with what obtained in the previous example using full matrices in term of non-zero rows. Since in this study, rules are driven from data set using fuzzy c-mean clustering method, above efficient approach is utilised.

3-4-Fuzzy Utility Models

There are some drawbacks with RUMs in discrete choice modelling such as dealing with qualitative and linguistic variables, and incorporating fuzzy logic has been suggested as promising approach (Lien & Chen, 2002). Different approaches have been considered by different scholars to utilise fuzzy logic in RUMs, however no fuzzy utility models (FUMs) based on the specific theoretical paradigm - for travel behaviour - have been proposed (Quattrone & Vietetta, 2011). In this section couple of studies that FUMs have been used through different approaches are reviewed.

Koutsopoulos and Lotan (1993b; 1993a) in their studies described an alternative formulation of the discrete choice problem, for the case of route choice in the presence of information. They modelled the decision makers' perceptions of the attributes of the various alternatives using fuzzy sets and the decision process using concepts from approximate reasoning and fuzzy control (Zadeh, 1976). They argued that drivers, especially in the context of making decisions under real time provision of information, do not compare alternatives in terms of exact values of their attributes. Instead they characterize the attributes of the alternatives they face using ‘‘linguistic’’ values such as high, very high, low, etc. (e.g. ‘‘travel time is high’’). Furthermore, to make a decision, they use a few simple rules that relate their vague perceptions of the various attributes to their preferences towards the available alternatives.

Mizutani and Akiyama (2001) studied a hybrid model “fuzzy logit model” of combining stochastic models based on random utility models and fuzzy reasoning models in order to create advanced models for the estimation of travel behaviour. In their proposed models, the utility function in the fuzzy logit model was described using fuzzy reasoning instead of using mathematical functions such as a liner function in order to reflect human decision with vagueness into discrete choice models. They proposed two methods for finding the utility functions, one using trial and error and the other one using Genetic Algorithm.

Vytoulkas and Koutsoplous (2003) extended the framework developed by Lotan and Koutsopoulos (1993b; 1993a) to incorporate rule weights which capture the importance of a particular rule in decision process. Rules weights showed their influence in composition stage in the proposed framework before. By using the concepts from the theory of neural networks, they could develop a fuzzy neural network procedure to calibrate the weights. They examined their developed model in a mode choice case study (rail or car) to evaluate it. Based on indicators of performance model, they found out that the results from the fuzzy model compare favourably to conventional choice models (Logit). It is argued that no completely specified and calibrated model was used in those studies.

In context of rout choice problem, other studies have been undertaken. In those studies, the systematic part of utilities were presented using fuzzy numbers or membership functions and comparison of these attributes. The chosen rout is based on the context of possibility theory and the same concept of RUM (Henn, 2000, 2003; Henn & Ottomanelli, 2006). The main issue with those researches were the poor representation of the fuzziness related to the perception process and construction of membership functions of fuzzy sets.

In terms of fuzzy utility models (FUM) and choice models, Quattrone and Vitetta (2011) validated and calibrated a rout choice model distinguishing the phases of set perception and rout choice among the alternatives belonging to the chosen set. They assumed that the systematic part of utility functions are obtained from different attributes and utility represents the core value of the associated fuzzy number (membership function) in form of a triangular distribution. For each alternative, the systematic utility (core of the triangular membership function) was calculated and using Minimum Squared Error Method (MSEM) the model was calibrated. The possibility of each alternative was obtained as a portion of that alternative's utility to summation of all alternatives' utilities. The effectiveness and applicability of the model was compared with a Multinomial Logit model.

Mamdoohi et al. (2014) incorporated fuzziness into random utility models in modelling the suitability of telecommuting from only job tasks point of view. In their research, aggregate variables of positive and negative time spent on job tasks were fuzzified using exponential membership functions and incorporated into binary logit model to study the fuzziness of this values. The results indicate that fuzzy models performed better than non-fuzzy binary logit model.

Different approaches in using fuzzy logic in discrete choice modelling were reviewed. As mentioned before, there is no specific paradigm for using fuzzy logic in developing choice models (Quattrone & Vietetta, 2011) and different scholars have implemented different methods. In all the studies above, black box models were presented and only inputs and outputs were considered. There is no approach – to the best of author's knowledge – that intermediate variables are taken into account and their influence on output reveals.

3-5-Discrete Choice Modelling Using Fuzzy Network

In this study and for modelling suitability of telecommuting, a comprehensive conceptual framework proposed by Mokhtarian and Salomon (1994) is used. In this framework, not only external inputs are important, but also intermediate variables have an influence on the output. It can also be inferred that outputs are influenced by functional nonlinearity. Besides, data are based on employee's estimation and attitudes which may be characterised by imprecision, incompleteness and vagueness. These characteristics are often referred to uncertainty.

Fuzzy logic is implemented in this study due to the capturing uncertainty of subjective knowledge and also facilitating common sense reasoning with imprecise and vague propositions dealing with natural language. Moreover, Rule based network as a hybrid between Single Fuzzy System (SFS) and Chained/Hierarchical Fuzzy Systems (CFS/HFS) has advantages of both systems.

RBN is implemented in this research for modelling suitability of telecommuting as a choice model. In other words, considering all the fundamentals and requirements of choice theory, rule based network is implemented to find the choice probability of each alternative. All the attributes of the fuzzy set theory is applied for dealing with humanistic uncertainty. Also, as a white-box modelling approach, RBN is well suitable for a modelling complex system which interactions between external and intermediate variables are important to be considered.

3-5-1-Fuzzy utility modelling using rule based network

As discussed in Section 2.3 and in choice theory, the probability of choosing an alternative depends on the utility of each alternative. Rational decision makers try to maximise their utility through their choice or alternative. In general, the random utility of an alternative is expressed as additive linear functions and sum of observable and disturbance components. It has also been mentioned that there is no imposed functional from for utility functions but convenient computational properties are required.

IF – THEN or rule based form is the most common form that represents human knowledge. It typically expresses an inference that if decision maker knows a fact (premise, hypothesis, antecedent), then another fact called conclusion (consequent) is derived. This form of knowledge, is called shallow knowledge, is quite appropriate in the context of linguistics because it expresses human empirical and heuristic knowledge in the language of communication. The fuzzy rule based system is most useful in modelling some complex systems that can be observed by humans because it makes use of linguistic variables as its antecedents and consequents (Ross, 2004).

In a multidimensional decision problem like suitability telecommuting, various variables are representing different aspects of employees' work and life like family, job responsibilities, commuting and so on. It means that employees deal with a number of linguistic terms and variables to have an understanding of each alternative's utility. Rule based system is considered for representing employee's subjective knowledge. Moreover, as conceptual framework shows, there are intermediate variables which influence on the output. These intermediate variables are fed by external inputs and function as a rule based system which should be represented as modular rule bases that reflect the subsystems.

Rule based network is a very promising approach that addresses most of the issues discussed above. Therefore, in this study, using the telecommuting framework, the appropriate network is driven to represent external inputs, intermediate variables, their connections and interactions and also relevant output. The utility of each alternative is calculated using linguistic composition and sequence of Fuzzification, Inference and Defuzzification over an equivalent single fuzzy system.

Different approaches have been discussed It was shown in chapter 2 to obtain the utilities of alternative for choice models. Systematic component of utility function has been obtained from fuzzy numbers or membership function (Quattrone & Vietetta, 2011). In this study the systematic part of random utility is obtained using RBN and to maximise the utility of each alternative, the fuzzy numbers should be maximised which is obtained using Genetic Algorithm (will be explained in the next section). Having the maximum utility of each alternative, the probability/possibility of each alternative is obtain using "strict utility" as (Klir, 1990; Luce, 1959; Tversky, 1972) proved:

$$P(i|C_n) = \frac{U_{in}}{\sum_{j \in C_n} U_{jn}} \tag{54}$$

Where

U_{in}: Utility of alternative *i* for decision maker *n*

$P(i|C_n)$: Probability of alternative *i* is chosen from choice set C_n

Generally speaking, in utility functions, there are different variables which are representative of various factors that influence the utility of alternatives and also parameters that adjust the weighting of each variable in utility function in an additive format. In RBN model which works on Mamdani method (Gegov, 2007), utility functions are nonlinear and the connection of antecedents are conjunctive which represents the overlap between variables that could be even stronger situation than additive. Also, membership functions are representative of the parameter that gives weights to variables based on given values. Having the maximum utility for each alternative, the probability choice for each alternative is obtained using the mentioned approach.

3-5-2-Implementing Genetic Algorithm for Utility Maximization

In choice models with linear utility functions as mentioned in Section 3-2-5, unknown parameters (β) are estimated for which observed sample is most likely to occur. Maximum Likelihood and Least Square are popular methods for estimating parameters. Having estimated parameters, maximum utilities of alternatives can be obtained for individuals and finally choice probability of each alternative is worked out.

In fuzzy set theory and RBN model, there is no unknown parameter. So, methods such Maximum likelihood cannot be utilised. On the other hand, membership functions act similar to parameters in functions having coefficients for variables (Ross, 2004; Shapiro, 2005). The same as what Maximum likelihood does, membership functions should be optimised and tuned in order to maximise the utility of each alternative and make the observed sample most likely occurred.

To tune fuzzy systems and optimise membership functions and rules, various methods have been discussed in the literature and MATLAB software is also widely used. MATLAB has specific Fuzzy Logic Tool-Box application that helps modelling data using the fuzzy set theory. It also offers adaptive techniques like Adaptive Neuro-Fuzzy Inference Systems (ANFIS) which create a fuzzy system to match any set of input-output data. The neuro-adaptive learning method works similarly to that of neural networks. Neuro-adaptive learning techniques provide a method for the fuzzy modelling procedure to learn information about a data set.

ANFIS constructs a fuzzy inference system (FIS) and uses a hybrid learning algorithm to tune the parameters of a Sugeno-type fuzzy inference system (FIS). The algorithm uses a combination of the least-squares and backpropagation gradient descent methods to model a training data set. ANFIS also validates models using a checking data set to test for overfitting of the training data. This adjustment allows fuzzy systems to learn from the data they are modelling (Roger & Gulley, 1995).

As just mentioned above, ANFIS works only for Sugeno-type fuzzy inference systems. However, rule based networks are based on Mamdani-type fuzzy systems. In this case, ANFIS, which is a powerful tool for tuning fuzzy systems and modelling data, cannot be implemented.

To tackle the above-mentioned issue, another function of MATLAB can be useful. GENFIS3 generate Mamdani/Sugeno fuzzy inference system using fuzzy c-mean (FCM)[1] clustering by

[1] Fuzzy c-means (FCM) is a data clustering technique wherein each data point belongs to a cluster to some degree that is specified by a membership grade. This technique was originally introduced by Jim Bezdek (1981) as an improvement on earlier clustering methods. It provides a method that shows how to group data points that populate some multidimensional space into a specific number of different clusters. FCM starts with an initial guess for the cluster centres, which are intended to mark the mean location of each cluster. The initial guess for these cluster centres is most likely incorrect. Additionally, FCM assigns every data point a membership grade for each cluster. By iteratively updating the cluster centres and the membership grades for each data point, FCM iteratively moves the cluster centres to the right location within a data set. This iteration is based on minimizing

extracting a set of rules that models the data behaviour. Actually, the purpose of clustering is to identify natural groupings of data from a large data set to produce a concise representation of a system's behaviour. The function requires separate sets of inputs and output as input arguments. When there is only one output, GENFIS3 can generate an initial FIS for training. The rule extraction method first uses the FCM function to determine the number of rules and membership functions for the antecedents and consequents (Roger & Gulley, 1995). Also, the rules are found by clustering and not by dividing the input space into grid-like partitions, this method can generate good models that contain only a small set of rules irrespective of the dimension of the data (Chiu, 1996).

In this research, GENFIS3 is used to obtain the initial model (rules and membership functions) based on fuzzy c-mean. To simplify the problem, 3 clusters are considered for each modular rule based systems. In another word, 3 linguistic terms and membership functions are generated for each variable and 3 rules are driven from data for each subsystem.

The next step is to tune the initial FIS the same as what ANFIS does for Sugeno fuzzy-type systems. To tune the fuzzy model with having 3 rules for each rule based subsystem, membership functions should be developed and optimised.

To develop membership functions different methods have been proposed in the literature such as Intuition, Inference, Rank Ordering, Neural Network, Genetic Algorithms and Inductive Reasoning (Ross, 2004). Among all the existence methods, Genetic Algorithm (GA) has proved itself as a transparent, interpretable and convenient adaptive search technique based on natural selection and genetic rules.

GENFIS3 generates Gaussian membership functions for Mamdani fuzzy-type system which is slightly challenging to get optimised. As there is no obligation on the shape of membership functions, for sake of simplicity shape triangular membership function are approximated for this study as Figure 3.9 shows.

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an objective function that represents the distance from any given data point to a cluster centre weighted by that data point's membership grade.

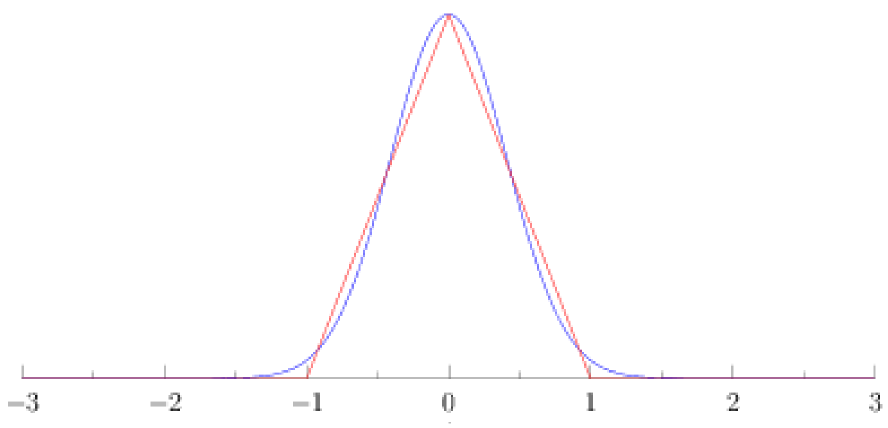


Figure 3.9: Approximating Gaussian MF with triangular MF

The first step in GA is constructing an initial population as a potential solution which is generalised randomly or heuristically. The final solution of GA is coordinates of all vertices for all triangular membership functions. In this research, the initial population size is about *10*N* genes where N is the number of variables (inputs and output) in each modular rule based. Initial population consist of two types of chromosomes, N-1 are randomly constructed and the other one is an approximation of the Gaussian membership functions (Genfis3 output) with simple triangular membership functions. The reason to put the approximated solution from Genfis3 into the initial population is to provide a chance for search technique to reach to the optimum point faster and speed up convergence. Also, positions of vertices for each membership function are in binary encoding with the length of 6 bits.

The next step is to assess the performance, or fitness, of individual members of a population. This is done through an objective function that measure of how individuals have performed in the problem domain. The objective function establishes the basis for selection of pairs of individuals that will be mated together during reproduction. The fitness function is normally used to transform the objective function value into a measure of relative fitness.

During the reproduction phase, each individual is assigned a fitness value derived from its performance measure given the objective function. In optimising membership function in this study, the objective function is a minimization problem which tries to minimise the difference between observation and the output of fuzzy inference system. Thus, the lower objective function values correspond to fitter individuals (Arslan & Kaya, 2001; Ross, 2004).

In this research and using MATLAB Genetic Algorithm Toolbox (Chipperfield, Fleming, Pohlheim, & Fonseca, 1994), not to get negative values for minimising problem, values obtain from objective functions are ranked. In linear ranking approach, the fitted individual is given 2 and the least fit value of 0. Then, the relative fitness (probability) of an individual being selected is determined by comparing the given index number $f(x_i)$ with the cumulative sum of indices.

$$F(x_i) = \frac{f(x_i)}{\sum_{i=1} f(x_i)} \tag{55}$$

Selection is the next step. When relative fitness values are obtained, parents are selected. Selection is the process of determining the number of times a particular is chosen for reproduction and, thus, the number of offspring that an individual will produce. Using Stochastic Universal Sampling (SUS) (Baker, 1987),which embedded in MATLAB Genetic Algorithm Toolbox, the selection is taken place and reproduction happens.

By choosing the individuals from the population and recombining them, new chromosomes are produced. The basic operation for producing new chromosomes is Crossover. Like its counterpart in nature, crossover produces new individuals that have some parts of both parents' genetic materials. The simplest form of crossover is single-point crossover which is achieved in three stages. First matching and selecting two individual randomly, then crossover point is determined in each individual and finally two parts of the individuals are replaced with each other. Crossover is not necessarily performed in all strings in the population and it is applied with a probability when the pairs are chosen for breeding (This probability in this study is 0.7).

For faster convergence, fewer individuals are produced by recombination than the size of the original population. In the other words, usually, the least fit members are replaced by the fit ones. The fractional difference between the new and old population sizes is called the generation gap. Generation gap, in this study, is 0.9 which means 10 percent of the population is removed in each iteration. In return, most fit individuals are deterministically allowed to propagate through successive generation. This step is called Reinsertion.

Mutation is undertaken for the next step. It is applied to the new individuals with a given probability, which is assumed 0.7 in this study and causes a single bit to changes its state from 0 $\longrightarrow$ 1 or 1 $\longrightarrow$ 0. Mutation is generally considered to be a background of the operator that ensures that the probability of searching a particular subspace of the problem space is never zero (Chipperfield et al., 1994).

In the next step, the performance of the population is measured using the objective function. Over a number of iterations, the GA tries to minimise the objective function and produce new membership functions. As the GA is a stochastic search method, it is difficult to specify convergence criteria. So, the termination of GA in this study is defined either for 1000 iterations or getting 50 times the same value for the fittest individual.

Using the above approach, GA, optimised membership functions are obtained and FIS is tuned. For tuning the whole model or the fuzzy network, it is required to tune each modular subsystems using Genfis3 for extracting rules and GA for optimising membership functions.

3-6-Input and Branch Selection

One of the great challenges in modelling nonlinear systems is selecting the important input variables from all possible inputs. From a modelling perspective, incorporating only the important variables into a model provides a simpler, more useful, and more reliable model; the model will also be more practical to apply because fewer variables need to be measured. From a control perspective, understanding the relative importance of variables allows the process control engineers to focus their efforts on the variables that matter, eliminating the time and cost involved in controlling and finding good set-points for unimportant variables (Chiu, 1996).

Guyon and Elisseeff (2003) in their study did a comprehensive review over different methods in variable selection. The objective of variable selection was mentioned as improving the prediction performance of the predictors, providing faster and more cost-effective predictors and providing a better understanding the underlying process that generated the data.

Concerning Random Utility Models, correlation analysis (Cohen, Cohen, West, & Aiken, 2013) between inputs and outputs are performed. Those independent variables which are less correlated to the output and also highly correlated with other independent variables are removed from the data set for modelling. Moreover, in model calibration process, those coefficients who are not statistically significant are removed from the model.

For fuzzy models, statistically rigorous criteria for model selection do not exist and heuristics criteria are relied. Even when a good criterion exists for model selection, there is no guarantee that a model based on a given set of variables is optimal unless all possible combinations of variables have been explored (Chiu, 1996).

Different methods have been proposed in the literature for input selection for fuzzy simple models such as forward and backward selection methods which are computationally expensive, required many different models to be generated while exploring different combinations for variables (Hayashi, Nomura, Yamasaki, & Wakami, 1992; Tanaka, Sano, & Watanabe, 1995). Also, principal component analysis using eigenvectors of input variables is used though is not reliable for input variables for large variance (Yen, Wang, & Liao, 1993). Implementing Genetic Algorithm and Neural Networks as well-known heuristic optimisation approach has been used widely for input selection and classification. However, they are iterative methods and number of models should be developed while searching for the optimal combination of variables in either approaches.

An established, efficient and fast method which overcomes computational bottlenecks (the need to generate new models to test each combination of variables) was proposed by Chen (1996). In this backward selection approach, the tuned fuzzy model first is obtained using all the inputs in training data set. Then inputs are systematically removed using testing data set to search for the beset simplified model without actually generating any new models.

One of the main challenges of the Rule Based Network is dimensionality. In large data sets with large number of input variables, like telecommuting, developing model is computationally burdensome. Not only important external inputs should be identified, but also those branches of the network which do not play an important role in describing the output should be removed to decrease the size of the network and consequently increase the efficiency. There is no existing method for input and branch selection in the literature and appropriate approach should be considered for addressing this challenge.

In a complex model and network, one of the major issues is dimensionality. In order to reduce the size of the model and make it more practical, inputs variables should be screened. Those input variables that have no or less influence on output can be removed from the model and the combination of important variables should be kept. Being important is interpreted statistically or in a way to improve the accuracy, transparency and efficiency of the model.

Different approaches have been studies and widely use in modelling. In this study, two methods are applied for selecting the optimal combination of variables: correlation analysis and forward selection for Multinomial logit model and also backwards combination approach for rule based network.

3-6-1-Correlation criteria

To have a whole picture of variables and their relationships, correlation analysis can reveal useful information. The most commonly used reliance measure in multivariate statistics is Pearson correlation. The Pearson correlation coefficient is defined between *x* and *y* as:

$$R(i) = \frac{cov(X_i,Y)}{\sqrt{var(X_i)var(Y)}} \tag{56}$$

Where *cov* designates the covariance and *var* the variance:

$$R(i) = \frac{\sum_{i=1}^{m}(x_{k,i}-\overline{x}_i)(y_k-\overline{y})}{\sqrt{\sum_{k=1}^{m}(x_{k,i}-\overline{x}_i)^2\sum_{k=1}^{m}(y_k-\overline{y})^2}} \tag{57}$$

Correlation criteria such as $R(i)$ can only detect linear dependencies between variables and target. But, identification of significant correlations is a common technique in data mining. The selection of candidate variables that are sorted by order of decreasing correlation is based either on greedy selection of the first k variables, or upon all variables for which the correlation is significantly different from zero. The significance of the Pearson correlation can be determined directly, since the error associated with estimation of correlation from a sample is defined by the t-distribution (Zuwaylif, 1984).

In this research and for Multinomial Logit model, at the first step and using SPSS software the matrix of Pearson correlation is obtained. Then, the independent variables that have high correlation with output variable and also a low correlation between each other are nominated for feeding the model.

3-6-2-Forward selection for Multinomial Logit Model

As it is commonly used in MNL models, the potential set of input candidates (using correlation criteria) are used for developing the model. In a forward selection method, the model is run and calibrated using Maximum Likelihood method and each time the unknown parameters (β) are obtained. In each run, the statistically important variables using t-test (5% level of significance for calibrated unknown parameters and meaningful difference with zero) are kept in model and others are removed. In forward selection approach and by developing numerous models the best combination of input variables is selected and presented in the final and most fitted model.

3-6-3-Backward Selection Method for Fuzzy Network

To select the optimal combination of variables for nonlinear models such as fuzzy models, statistically rigorous criteria for model selection do not exist and must rely on heuristic approaches. In most of the heuristic approaches, new models should be repeatedly developed and evaluated to identify the optimal combination (Chiu, 1996).

In rule based network, which consists of subsystems, it would be computationally very heavy to develop and generate repeatedly models and select the important inputs for each subsystem. Also, after selecting the most important external inputs, for the next layer of rule based network model, intermediate variables act as inputs for the other layer which means again number of models should be developed subsequently over all subsystems. In addition, as explained in Section 3-6-2 and for tuning each model, genetic algorithm optimises the models and membership functions in an iterative approach which means it would be computationally so complex and expensive to incorporate both methods.

In this study a backward selection is used to address the mentioned issues. This method is based on generating only one fuzzy model that takes all the possible input variables and then systematically removing antecedent clauses in the fuzzy rules of this initial model to test the significance of each variable. In other words, this is essentially a backward selection procedure that avoids the need to repeatedly generate new models to test each combination of variables (Chiu, 1996).

Firstly, an initial fuzzy model that incorporates all the possible inputs should be generated and tuned using the training data. The idea is to simply remove all antecedent clauses associated with particular input variable from the rules and then evaluate the performance of the model using the checking data. The systematic procedure for input variable selection as follows:

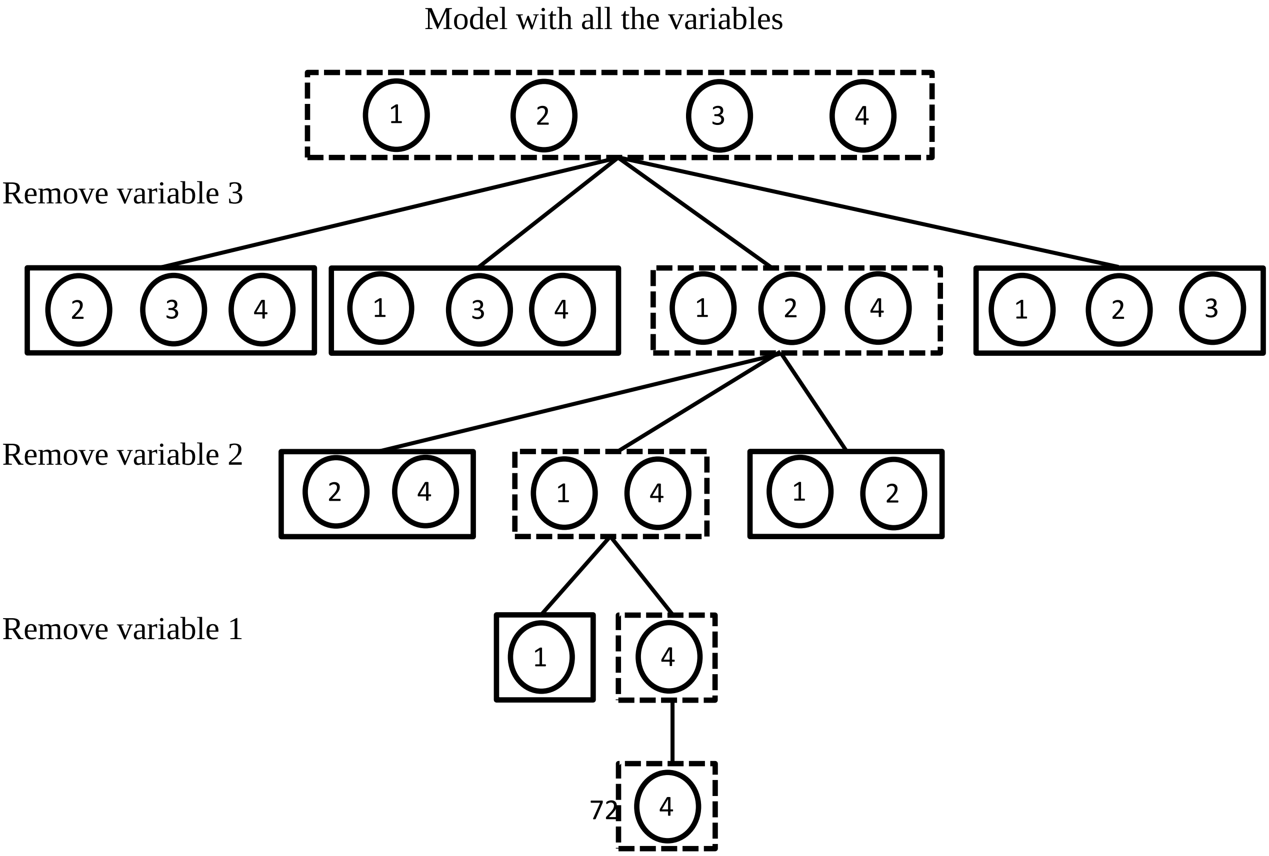
1- Evaluate the performance of initial model with all candidate input variables in the model using training data
2- For each remaining input variable, evaluate the performance of the initial model with this variable temporarily removed using checking data.

3- Permanently remove the variable associated with the best model performance obtained in step 2. Record the resultant reduced variable set and the associated model performance.
4- If there are still variables remaining in the model, go back to step 2 to eliminate another variable. Otherwise go to step 5.
5- Choose the best variable set from among the sets recoded in the step 3.

The variable selection process of a four-input initial model is shown in Figure 3.10 to illustrate the proposed approach.

Figure 3.10 shows different subsets of variables considered at each stage. The dashed subsets are the ones that provided the best performance at each stage. After the variable removal process is carried to the end, the best performance is selected. To highlight the rule of human judgment and design in this variable selection process, it is useful to carry the process to the end with a model with no input variable. The trade-off between accuracy and simplicity play an important role in input selection.

A model selection criterion is needed to select the best performance. The performance criterion is the model's Root Mean Square (RMS) output error with respect to an independent set of checking data. The data is divided into two groups: training (A) and checking (B) data set. The initial model is generated and tuned using training data (A).



Model with no input variable

Figure 3.10: Variable selection process for a four-input initial model

The checking error criterion is then defined as:

$$J_c = \sqrt{\frac{\sum_{i=1}^{n_B}(z_i^B - z_i^{BA})^2}{n_B}} \tag{58}$$

where z_i^B is the i^{th} output data in group B and z_i^{BA} is the corresponding predicted output for group B obtained from trained model using A and n_B is the number of data points in group B. This criterion is evaluated repeatedly as each input variable is removed from trained model and choose the variable subset that produced the smallest J_c at each stage.

In rule based network model in this study, the data is divided into two groups. For each subsystem in the layer 1 and considering its external inputs, the initial model is developed, tuned and optimised using Genfis3 function of MATLAB and genetic algorithm. Then, using the backward selection method, the optimal combination of variables is selected. The outputs of selected models for subsystems in layer 1 act as input for intermediate nodes in later 2. By duplicating this approach for the rest of layers and subsystems, inputs and intermediate variables are selected. In fact, not only external inputs are selected but also those important branches of the network and intermediate variables are kept in the final model and less important ones are removed to minimise the size and dimension of the rule based network.

In the next step, having optimised membership functions, data-driven rules, selected inputs and intermediate variables and by using linguistic composition approach described in Section 3-5-2, the final manipulated model is obtained. Only by performing a single Fuzzification, Inference and Defuzzification sequence for the whole model, the output, which is the utility of each alternative, is calculated.

3-7-Fuzzy Network Model Evaluation

The focus of the linguistic composition approach is to maintain accuracy by representing a HFS as a SFS with a single FID sequence while improving transparency by means of the modular rule bases that reflect the subsystems of the modelled system.

The quality of the rule based model can be quantified using performance indicators. In particular, three model performance indicators are introduced further below. They are called Accuracy Index (AI), Efficiency Index (EI) and Transparency Index (TI). These performance indicators represent modifications of performance indicators used for fuzzy systems that can also be used for fuzzy networks.

The first performance indicator AI reflects the accuracy of the model by means of the absolute difference between the model and the data, as shown in Equation (51)

$$AI = \sum_{i=1}^{nl} \sum_{j=1}^{qil} \sum_{k=1}^{v_{ji}} (|y_{jik} - d_{jik}| \,/\, v_{ji}) \tag{51}$$

The notations in Equation (27) are as follows: nl is the number of nodes in the last layer, qil is the number of outputs from the i-th node in the last layer, vji is the number of discrete values for the j-th output from the i-th node in the last layer, y_{ji}^k is the simulated k-th discrete value for the j-th output from the i-th node in the last layer and d_{ji}^k is the measured k-th discrete value for the j-th output from the i-th node in the last layer, '*sum*' is a symbol for arithmetic summation and '| |' is a symbol for absolute value. Identity nodes are included in this indicator alongside any other nodes in the last layer because their outputs also have to be compared with the data. As a model is more accurate when the absolute difference between the model and the data given by Equation (51) is smaller, a lower AI implies better accuracy.

The second performance indicator EI reflects the efficiency of the model by means of the overall number of rules, as shown by Equation (52)

$$EI = \sum_{i=1}^{n} (q_i^{FID} . r_i^{FID}) \tag{52}$$

The notations in Equation (28) are as follows: n is the number of non-identity network nodes, q_i^{FID} is the number of outputs from the i-th non-identity node with an associated FID sequence, r_i is the number of rules for the i-th non-identity node with an associated FID sequence and '*sum*' is a symbol for arithmetic summation. Identity nodes are excluded from this indicator because they are virtual nodes for converting a HFS into a FN that does not affect the efficiency. As a model is more efficient when the overall number of rules given by Equation (28) is smaller, a lower EI implies better efficiency.

The third performance indicator TI reflects the transparency of the model by means of the extent of its opaqueness from the inside, as shown by Equation (53)

$$TI = (p + q) / (n + m) \quad (53)$$

The notations in Equation (53) are as follows: *p* is the overall number of inputs, *q* is the overall number of outputs, *n* is the number of non-identity nodes, *m* is the number of non-identity connections and '*sum*' is a symbol for arithmetic summation. Identity nodes are excluded from this indicator as they are virtual nodes for converting a HFS into a FN that do not affect the transparency. As a model is more transparent when the extent of its opaqueness from the inside given by Equation (53) is smaller, i.e. the overall number of inputs and outputs is bigger while at the same time the number of sub-models and connections is smaller, a lower TI implies better transparency (Gegov, 2010) and [1].

3-8- Models Comparison

In this research the aim is to study the suitability of telecommuting using rule based network. To examine the performance of RBN, it is required to compare it with MNL as an existing approach. These two methods are totally different from various modelling perspectives, but to quantify their performances, the following indicators are also defined apart from their own evaluating techniques discussed in previous sections:

1- Accuracy: Accuracy of the models are examined using two approaches below:

   a) Percent correct: shows the accuracy of models in terms of correct predictions against the observation. The observation in this study is the number of the days that telecommuting is suitable for employees as they stated in the questionnaire and prediction is the highest probabale of alternatives obtained by model.

   b) Average distance between observation and model prediction:

$$A = \frac{|Y_i - y_i|}{n} \quad (59)$$

   Where

   Y_i: Choice observation of individual *i*

   y_i: Model prediction of individual *i*

   n: Total number of individual

2- Efficiency: Efficiency of models are assess from two perspectives:

a) Efficiency of the model by means of the overall number of rules as explained in equation 52. The problem with this indicator is that MNL cannot be assessed using that indicator.
b) Efficiency of the model by number of time that models are developed, tuned and calibrated.

For Rule based Network Model:

$$E_{RBN} = \sum_{j=1}^{m} \sum_{i=1}^{n} \left(\frac{x_{ij}(x_{ij}+1)}{2} - 1\right) + mn \tag{60}$$

Where

m: Number of layers

n: Number of levels

x_{ij}: Number of inputs in layer i and level j in non-identity nodes

For Multinomial Logit Model:

$$E_{MNL} = \frac{x(x+1)}{2} \tag{61}$$

where x is the number of inputs.

3- Transparency index: explained in equation 53
4- Interpretability index: Interpretability for fuzzy system has been discussed in the literature and shows how fuzzy models can be interpreted by human (Gacto, Alcalá, & Herrera, 2011).

For Rule based Network Model: interpretability of RBN can measure by number of the rules N_r, linguistic terms N_l, number of inputs N_x and number of outputs (alternatives) N_a. It can be quantify using following:

$$I_{RBN} = N_r.N_l.N_x.N_a \tag{62}$$

For Multinomial Logit Model: the ability to interpret the model confined to the number of variables N_x or their coefficients and also the number of alternatives N_a :

$$I_{MNL} = N_x.N_a \tag{63}$$

3-9- Conclusion

In this key chapter, the theoretical framework of this research were discussed and the research methodology was justified. As explained, adopting telecommuting is a multidimensional decision and employees use more linguistic and subjective approach rather than exact and precise values for choosing an alternative. Therefore, fuzzy logic is utilised as a powerful approach for capturing that type of uncertainty in choice modelling. Also, among different fuzzy system, Fuzzy Network was justified to be used due to its transparent nature for understanding the role of intermediate variables. Thus, a fuzzy utility model is developed using fuzzy network to find the utility of alternatives and fine the probability of choices.

In a data driven approach, model calibration is performed using Genetic Algorithm for tuning the fuzzy networks. Also, to tackle fuzzy network dimensionality issue and in order to find the most important external input and intermediate variables, input and branch selection method was proposed using backward selection approach. Moreover, an efficient approach was proposed for horizontal and vertical merging of modular bases to reduce computational expenses. At the end, various indicators were introduced to assess the performance of the models from different perspectives.

The main theoretical contributions of this research were discussed in details in this chapter: modelling suitability of telecommuting using fuzzy networks, adopting choice modelling using fuzzy rule based network, calibrating and tuning fuzzy networks models using Genetic Algorithm, input and branch selection for fuzzy networks and also proposing new approach for horizontal and vertical merging of Rule Based Networks.

In the next chapter, the data gathered data for testing the proposed methodology is described and analysis.

CHAPTER 4
STATISTICAL ANALYSIS OF RESEARCH DATA

4-1-Introduction

In chapter 3, the methodology for modelling telecommuting was discussed. To see whether the research approach is suitable for modelling telecommuting as a case study, numerical data is needed to be used, feed to the models and validated. In this chapter[2], the gathered data is discussed.

Data itself plays an important role in modelling. Analyst and modeller should have a clear view of what the data is and how it has been gathered. Interpreting the data is the prerequisite stage before using it in modelling. Blind modelling without understanding the data might lead to nonsensible and unreliable result. Thus, it is important to analyse the data and see the variation and diversity of variables. It also gives an overlook to the sample and would be helpful in different stages of modelling such as input and branch selection of the Fuzzy Network which was discussed in chapter 3.

In this chapter, firstly data gathering is explained, then the questionnaire is described to understand the type of questions have been asked and collected data. In the next section, data is analysed statistically with the help of tables and graph to give a better view to different variables such as job characteristics, sociodemographic factors, commuting variable, ideologies toward telecommuting and also suitability and preference for adopting telecommuting.

4-2-Data gathering

This data has been obtained from the questionnaire distributed at seven governmental and semi-governmental organizations in a central business district of Tehran, the capital city of Iran. As telecommuting was almost a new approach in terms of working, initially employees and their managers were given some information about it. The data consist of 245 individuals

[2] This chapter is parts of the collaborative research with professor M. Nadia Postorino at University of Reggio Calabria (awarded grant from European Cooperation in Science and Technology (COST))

and through a comprehensive interview socioeconomic and demographic factors, travel and commuting issues, preferences and suitability of adopting telecommuting have been asked from employees and their supervisors.

Employment rate in Tehran during the period of collecting this data was about 6-26 percent (Figure 4.1). As Figure 1 shows, in the central business district of this metropolitan, employment rate was in maximum range between 22-26 percent.

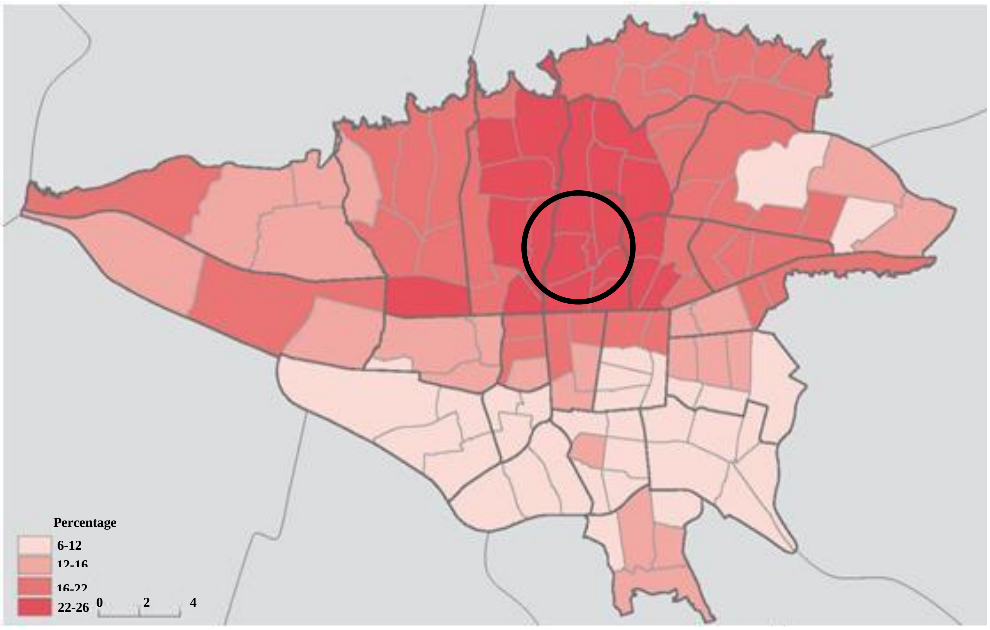


Figure 4.1- Employment rate in Tehran
Ref: Tehran Municipality official statistic website: www.atlas.tehran.ir

4-2-Questionnaire

Data has been gathered from employees and their employers through a questionnaire in different organizations. The questionnaire had been provided in 2 pages with 26 questions and it includes following information:

1) Organizations:
   - Computer Manufacturing Co.
   - Tehran Comprehensive Transportation and Traffic Studies (TCTTS)
   - Ministry of Interior
   - Tehran Municipality
   - Ministry of Roads and Transportation
   - Ministry of Agriculture

- Ministry of Economic Affairs and Finance

2) Job categories:

- Advert/Marketing/Sale/Customer Service
- Research/Consultation/Expertise
- Management/Supervision
- Programming/ Data Analysing/Computer Operating
- Production /Technical supporting
- Finance/Administrative/Accounting
- Secretariat/Clerical working/Secretary
- Supplying/Supporting/Services

3) Total work experience (years)

4) Work experience in current job (years)

5) Daily spending time on activities (employee's view): (Min/Hour)

- Reading/Writing reports
- Using Computer
- Using Phone or Fax
- Talking with Colleagues/Client
- Attending Meetings/Team working
- Working out of Organization

6) Daily spending time on activities (Supervisor's view): (Min/Hour)

- Reading/Writing reports
- Using Computer
- Using Phone or Fax
- Talking to Colleagues/Client
- Attending Meetings/Team working
- Working out of Organization

7) Importance of facilities in career (Employee's view):

- Fax/Phone
- PC/Software
- Reports/Correspondence
- Photocopier

- Special Places (Lab/Studio...)

8) Importance of facilities in career (Supervisor's view): (Low/Middle/High)

- Fax/Phone
- PC/Software
- Reports/Correspondence
- Photocopier
- Special Places (Lab/Studio...)

9) Gender

10) Age

11) Marital status

12) Household size

13) Number of children

14) Education:

- Diploma
- BSc (BA)
- MSc (MA) or PhD

15) Holding driving license

16) Car ownership

17) Commuting distance (Km)

18) Commuting issues from Home to Work:

- Mode choice
- Travel cost
- Travel time

19) Commuting issues from Work to Home:

- Mode choice
- Travel cost
- Travel time

20) Employees' ideologies on the impact of telecommuting on: *(No/Low/Middle/High)*

- Family psychic health
- Reducing traffic congestion
- Work efficiency

21) Number of days of doing telecommuting (Suitable/Reasonable):

- From home (Employee’s view)

- From home (Supervisor’s view)

22) Number of days of doing telecommuting (Preference):

- From home (Employee’s view)

4-3-Data Analysis

The collected data is analysed in this section, and related figures and tables are presented to give a picture of the sample.

4-3-1- Job characteristics

Data has been gathered from seven different organisations as follows in Table 4.1 and Figure 4.2:

Table 4.1- Share of organisations in sample

	Organization	Frequency	Percentage
1	Computer Manufacturing Co.	29	11.8
2	Tehran Comprehensive Transportation and Traffic Studies Co.	26	10.6
3	Ministry of Interior	50	20.4
4	Tehran Municipality	47	19.2
5	Ministry of Roads and transportation	51	20.8
6	Ministry of Agriculture	22	9.0
7	Ministry of Economic Affairs and Finance	20	8.2
8	Total	245	100.0

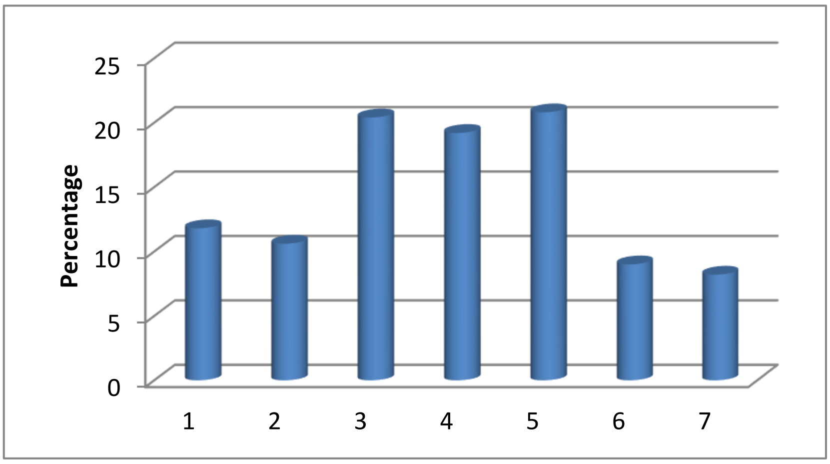


Figure 4.2- Percentage of sample’s employees in organizations (1-7 in X-axis represent the organizations in Table-4.1)

As can be seen from Table 1 and Figure 4.2, more than 60 percent of interviewees were from 3 organisation which might cause bias.

Job categories are based on their characteristics and have been divided into 9 groups. Table 4.2 and Figure 4.3 show job disparities.

Table 4.2- Job categories

	Job Category	Frequency	Percentage
1	Advert/Marketing/Sale/Customer Service	18	7.3
2	Research/Consultation/Expertise	103	42.0
3	Management/Supervision	39	15.9
4	Financing/Administrative/Accounting	17	6.9
5	Programming/ Data Analysing/Computer Operatory	29	11.8
6	Supplying/Supporting/Services	15	6.1
7	Producing /Technical supporting	6	2.4
8	Secretariat/Clerical working/Secretary	18	7.3
9	Total	245	100.0

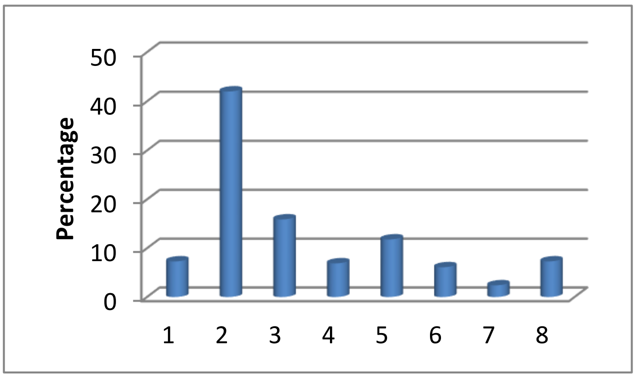


Figure 4.3- Percentage of employees in job categories (1-8 in X-axis represent the job categories in Table- 4.2)

From above figure, it can be inferred that the majority of individuals who have been interviewed are researchers and experts (about 40 percent of sample) which can also cause bias and skewness in results.

Total and current employees' work experiences a shown below.

Table 4.3- Average Employees' Work Experiences

	Total Experience (years)	Current Job Experience (years)
Work experience	10.1	5.4

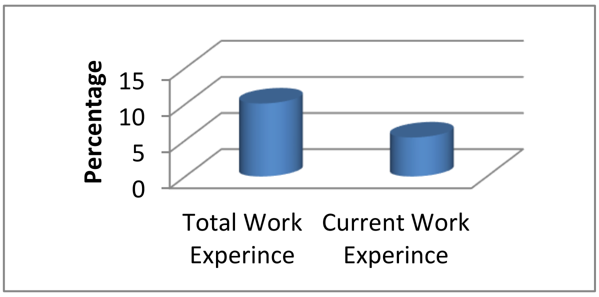


Figure 4.4- Average Employees' Work Experiences

The concept of abstract job (Mamdoohi et al. ,2006) has been considered in this research. The most important employees' tasks in their jobs have been asked from them and also from their supervisors. The average daily spent times on each activity are in Tables 4.4 and 4.5.

Table 4.4- Average Daily Spending Time on Activities – Employees' view

	Activity	Average daily spending time														Total	
		0 - 15 Min		15 - 30 Min		30 - 60 Min		1 - 2 Hours		2 - 3 Hours		3 -4 Hours		More than 4 Hours			
		Fre.	%	Fre.	%	Fre.	%	Fre.	%	Fre.	%	Fre.	%	Fre.	%	Fre.	%
1	Reading/Writing reports	23	9.4	32	13.1	41	16.7	69	28.2	44	18.0	20	8.2	16	6.5	245	100.0
2	Using Computer	55	22.4	23	9.4	32	13.1	37	15.1	24	9.8	25	10.2	49	20.0	245	100.0
3	Using Phone /Fax	85	34.7	60	24.5	44	18.0	34	13.9	9	3.7	5	2.0	8	3.3	245	100.0
4	Talking with Colleagues/Client	36	14.7	46	18.8	61	24.9	50	20.4	34	13.9	8	3.3	10	4.1	245	100.0
5	Attending Meetings/Team working	65	26.5	48	19.6	53	21.6	42	17.1	26	10.6	8	3.3	3	1.2	245	100.0
6	Working out of Organization	105	42.9	33	13.5	29	11.8	41	16.7	14	5.7	10	4.1	13	5.3	245	100.0

Table 4.5- Average Daily Spending Time on Activities – Supervisors' View

	Activity	Average daily spending time														Total	
		0 - 15 Min		15 - 30 Min		30 - 60 Min		1 - 2 Hours		2 - 3 Hours		3 -4 Hours		More than 4 Hours			
		Fre.	%	Fre.	%	Fre.	%	Fre.	%	Fre.	%	Fre.	%	Fre.	%	Fre.	%
1	Reading/Writing reports	14	5.7	30	12.2	54	22.0	85	34.7	35	14.3	17	6.9	10	4.1	245	100.0
2	Using Computer	49	20.0	25	10.2	30	12.2	43	17.6	42	17.1	23	9.4	33	13.5	245	100.0
3	Using Phone /Fax	57	23.3	63	25.7	61	24.9	38	15.5	16	6.5	8	3.3	2	0.8	245	100.0
4	Talking with Colleagues/Client	24	9.8	59	24.1	71	29.0	55	22.4	28	11.4	6	2.4	2	0.8	245	100.0
5	Attending Meetings/Team working	53	21.6	57	23.3	66	26.9	49	20.0	12	4.9	5	2.0	3	1.2	245	100.0
6	Working out of Organization	100	40.8	36	14.7	33	13.5	35	14.3	14	5.7	17	6.9	10	4.1	245	100.0

To have a better view to the job tasks, the time spent on each activity and also supervisors' view (form Tables 4.4 and 4.5) is drawn (Figure 4.5-4.10).

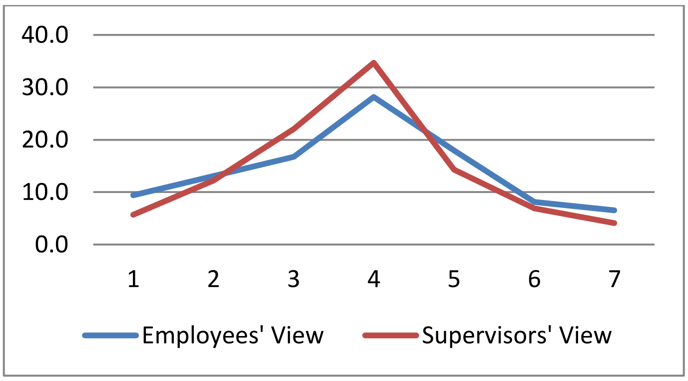


Figure 4.5- Average Daily Time Spent (percentage) on Reading/Writing reports (1-7 in X-axis represent alternatives)

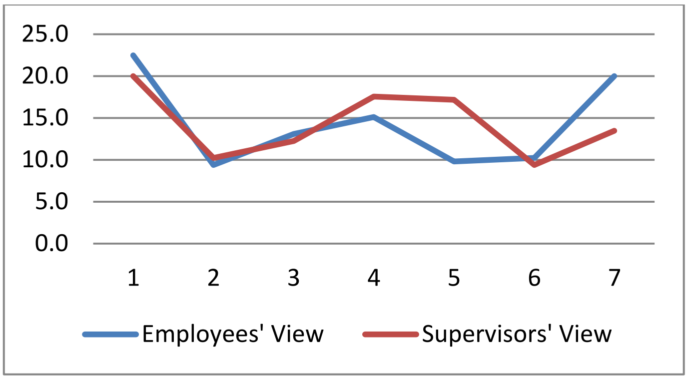


Figure 4.6 - Average Daily Time Spent (percentage) on Using Computer (1-7 in X-axis represent alternatives)

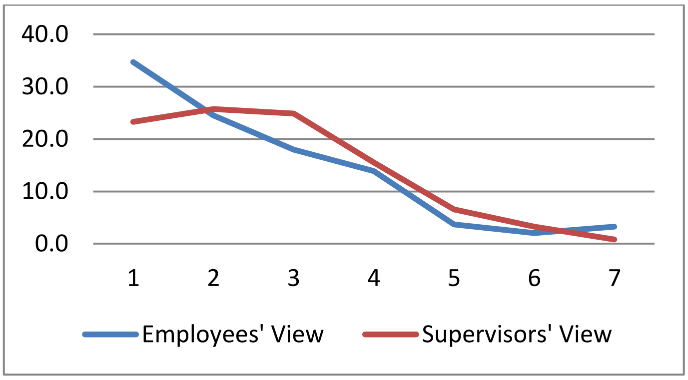


Figure 4.7- Average Daily Time Spent (percentage) on Using Phone/Fax (1-7 in X-axis represent alternatives)

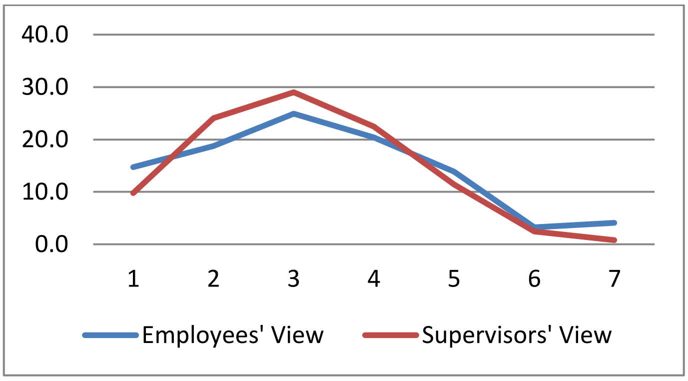


Figure 4.8- Average Daily Time Spent (percentage) on Talking with Colleagues/Client (1-7 in X-axis represent alternatives)

It can be seen from the Tables 4.4 and 4.5 and Figures 4.5-4.10 that employees and their supervisors' view are so close and there is no significant difference between them. Figures 4.5 demonstrates that about 63 percent of employees spent between 30 till 180 minutes on reading and writing reports and their supervisors' view is slightly more than that. Figure 4.6 reveals that about 30 percent spend more than 3 hours and on computer and 30 percent less than half an hour. Also Figure 4.7 shows that more than 70 percent of employees spent less than an hour on phone or fax.

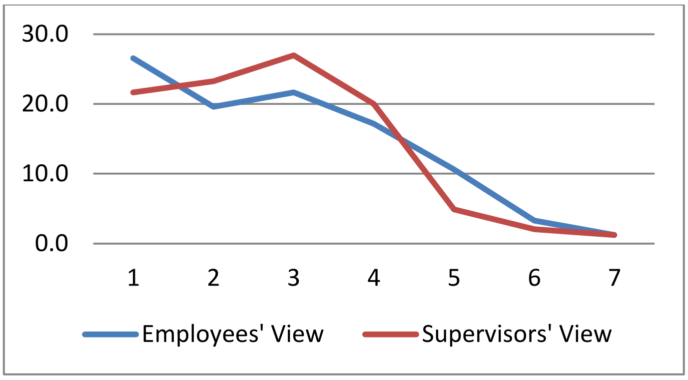


Figure 4.9- Average Daily Time Spent (percentage) on Attending Meetings/Team working (1-7 in X-axis represent alternatives)

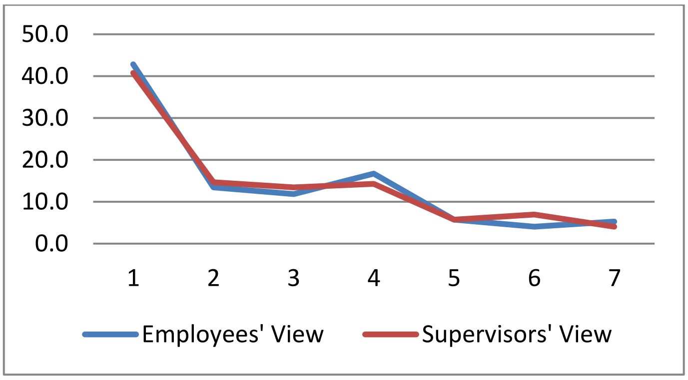


Figure 4.10- Average Daily Time Spent (percentage) on Working out of Organization (1-7 in X-axis represent alternatives)

In addition, Figure 4.8 discloses that about 60% of employees spend between 15 minutes till 2 hours on talking to colleagues or clients every day. Figure 4.9 shows that majority of the sample spend less than 2 hours on meetings/team working. Also, most of the employees work inside of their primary office and only 10 percent spend more than 2 hours outside of their offices for working purposes (Figure 4.10).

Facilities such as phone, fax, computer, reports and correspondences are important for employees. However, most of them are completely or partially independent from their preliminary office and by providing such facilities at home employees will be able to fulfil their responsibilities. Table 4.6 and 4.7 show the importance of different facilities from both employees and their supervisors' view. Also, Figures 4.11 – 4.15 show both employees and their supervisors' view towards importance of different facilities in employees' daily tasks.

Table 4.6- Importance of Facilities – Employees' view

	Facility	Importance						Total	
		Low		Middle		High			
		Fre.	%	Fre.	%	Fre.	%	Fre.	%
1	Importance of Fax/Phone	17	6.9	82	33.5	146	59.6	245	100
2	Importance of PC/Software	24	9.8	56	22.9	165	67.3	245	100
3	Importance of Reports/Correspondence	24	9.8	83	33.9	138	56.3	245	100
4	Importance of Photocopier	19	7.8	220	89.8	6	2.4	245	100
5	Importance of Special Places (Lab/Studio...)	155	63.3	58	23.7	32	13.1	245	100

Table 4.7- Importance of Facilities – Supervisor's View

	Facility	Importance						Total	
		Low		Middle		High			
		Fre.	%	Fre.	%	Fre.	%	Fre.	%
1	Importance of Fax/Phone	33	13.5	83	33.9	129	52.7	245	100
2	Importance of PC/Software	25	10.2	56	22.9	164	66.9	245	100
3	Importance of Reports/Correspondence	31	12.7	95	38.8	119	48.6	245	100
4	Importance of Photocopier	127	51.8	63	25.7	55	22.4	245	100
5	Importance of Special Places (Lab/Studio...)	152	62.0	56	22.9	37	15.1	245	100

Figure 4.11 shows clearly that phone/Fax is fairly important for employees and only less than 20 percent have found it with low importance.

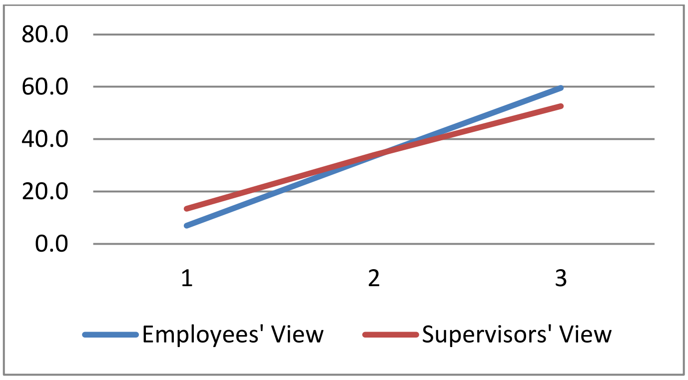


Figure 4.11- Importance of Fax/Phone (1-3 in X-axis represent alternatives)

As Figure 4.12 discloses, computers are counted as very important facility that employees and their supervisors have indicated that in questionnaire. It highlights the role of computers in daily jobs which needs to be considered for those who intend to adopt telecommuting.

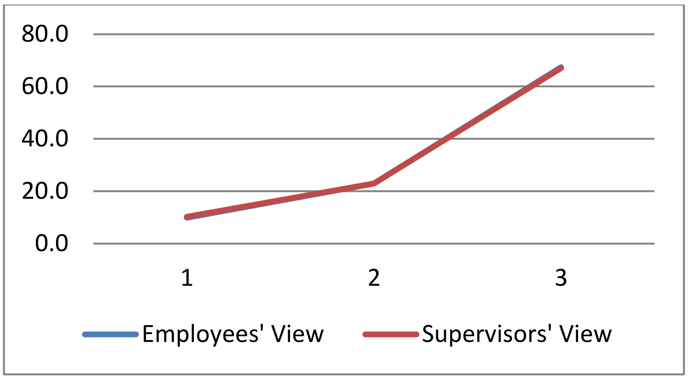


Figure 4.12- Importance of PC/Software (1-3 in X-axis represent alternatives)

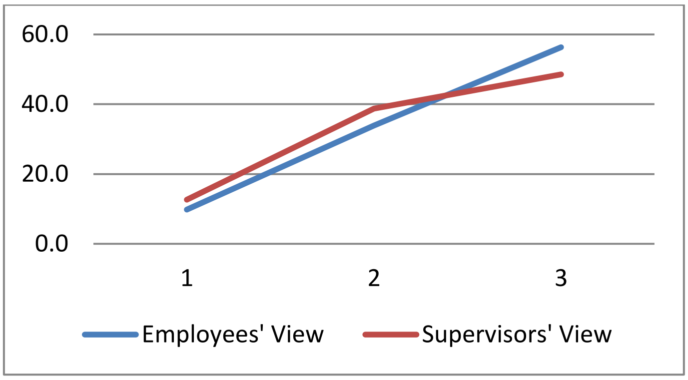


Figure 4.13- Importance of Reports/Correspondence (1-3 in X-axis represent alternatives)

Around 90 percent of employees have expressed the importance of reports and correspondence as middle or high (Figure 4.13).

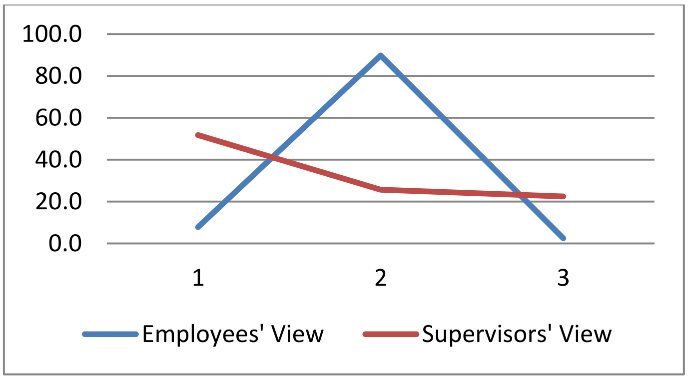


Figure 4.14- Importance of Photocopier (1-3 in X-axis represent alternatives)

For employees, photocopier machine has middle importance though it is not important from their supervisors' view (Figure 4.14) and special places like labs or studio is not important in this sample.

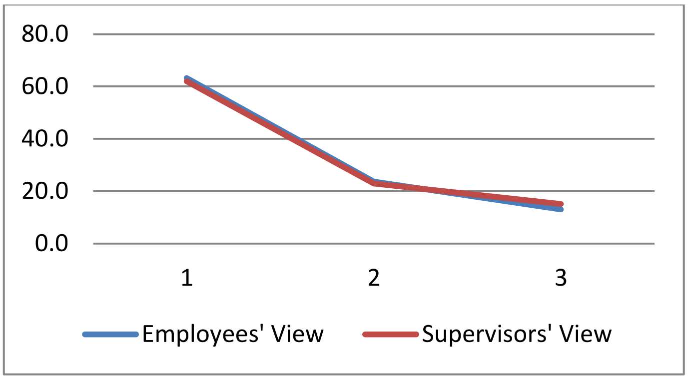


Figure 4.15- Importance of Special Places (Lab/Studio...) (1-3 in X-axis represent alternatives)

4-3-2- Sociodemographic factors

Demographic variables also play an important role in adopting telecommuting. Age is examined in 3 groups as Table 4.8 and Figure 4.16 show. This implies that age distribution is fairly uniform which leans more on over 40s.

Table 4.8 - Age groups

	Age Group	Fre.	%
1	<30	77	31.4
2	30-40	71	29.0
3	40<	97	39.6
4	Total	245	100.0

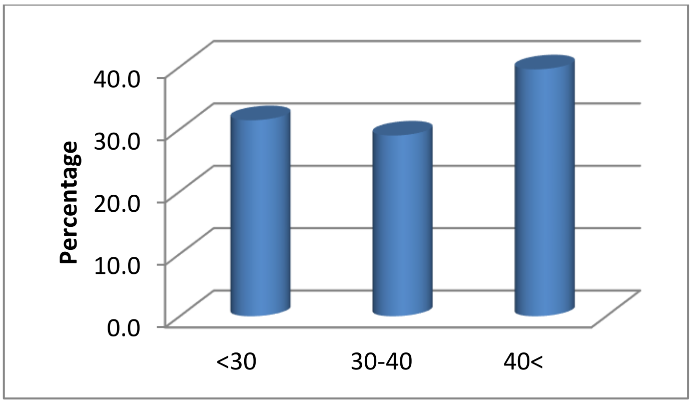


Figure 4.16- Age Groups (Percentage)

Gender is another important variable that should be studied. In this sample there are 175 men and 70 women. As it can be seen from Figure 4.17, about 70 percentage of the sample are men which it seems a bit bias.

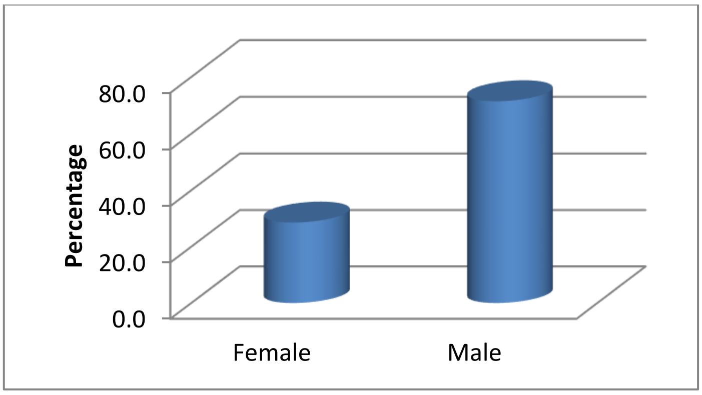


Figure 4.17 – Gender distribution

Marital status and household size are shown in Tables 4.9 and Figure 4.17 and 4.18. More than 60 percent of the sample is married and the average size of household is about 3.5 people per family.

Table 4.9 - Marital Status, Household Size

	Marital Status	**Household Size**										**Total**	
		1		2		3		4		>4			
		Fre.	%	Fre.	%	Fre.	%	Fre.	%	Fre.	%	Fre.	%
1	Single	9	3.7	8	3.3	13	5.3	26	10.6	28	11.4	84	34.3
2	Married	0	0.0	40	16.3	56	22.9	43	17.6	22	9.0	161	65.7
3	Total	9	3.7	48	19.6	69	28.2	69	28.2	50	20.4	245	100.0

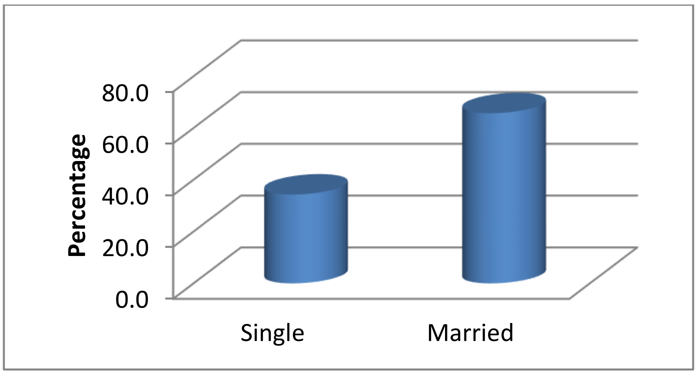


Figure 4.18- Marital Status (Percentage)

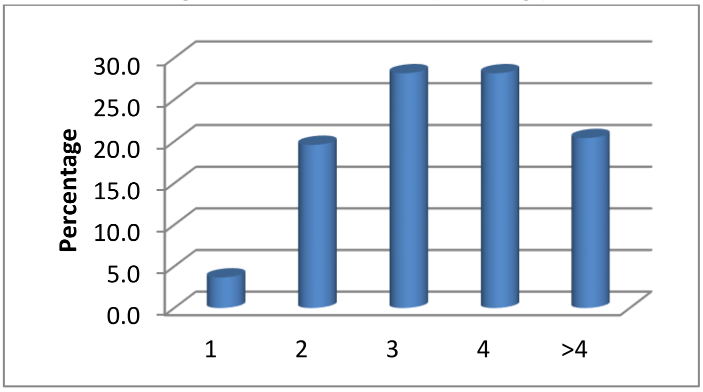


Figure 4.19- Household Size (Percentage)

Distribution of education level in the sample is shown in Table 4.10 and Figure 4.20 which show the majority of the sample hold bachelor degree.

Table 4.10 – Education level

Education	Diploma		BSc (BA)		MSc (MA) or PhD		Total	
	Fre.	%	Fre.	%	Fre.	%	Fre.	%
	59	24.1	121	49.4	65	26.5	245	100

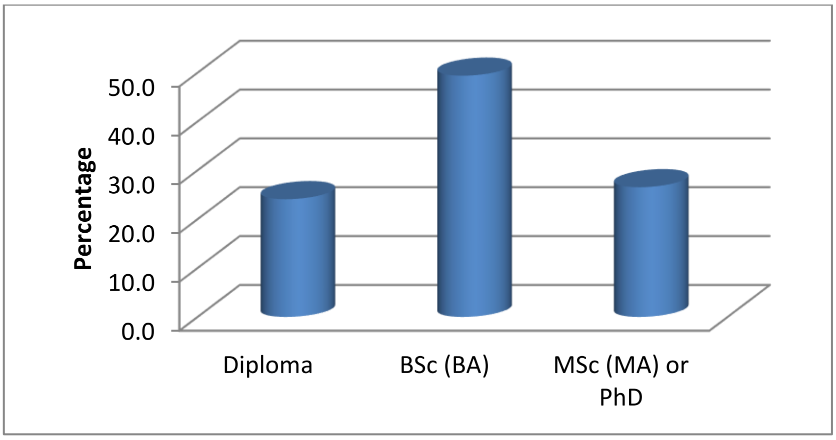


Figure 4.20- Education level (Percentage)

4-3-3- Commuting variables

Statistical analysis shows that 84 percent of the sample hold driving license and also 34 percent of participants have their own car. Also, in Table 4.11, commuting distance, travel time and cost from home to work and work to home have been summarised.

Table 4.11 – Travel issues

Travel Issues	Average for sample
Commuting Distance (km)	14.0
Travel Time: Home to Work (min)	45.2
Travel Time: Work to Home (min)	49.3
Travel Cost: Home to Work (min)	234.1
Travel Cost: Work to Home (min)	226.1

As literature shows, commuting issues play a crucial role in adopting telecommuting. Table 4.12 and Figure 4.21 show that public transport is the main mode of commuting in the studied sample.

Table 4.12 – Commuting Mode Choice

	Mode choice	Home-Work		Work-Home	
		Fre.	%	Fre.	%
1	Private Transport	63	25.7	60	24.5
2	Public Transport	174	71.0	179	73.1
3	Non-Motorized	8	3.3	6	2.4
4	Total	245	100.0	245	100.0

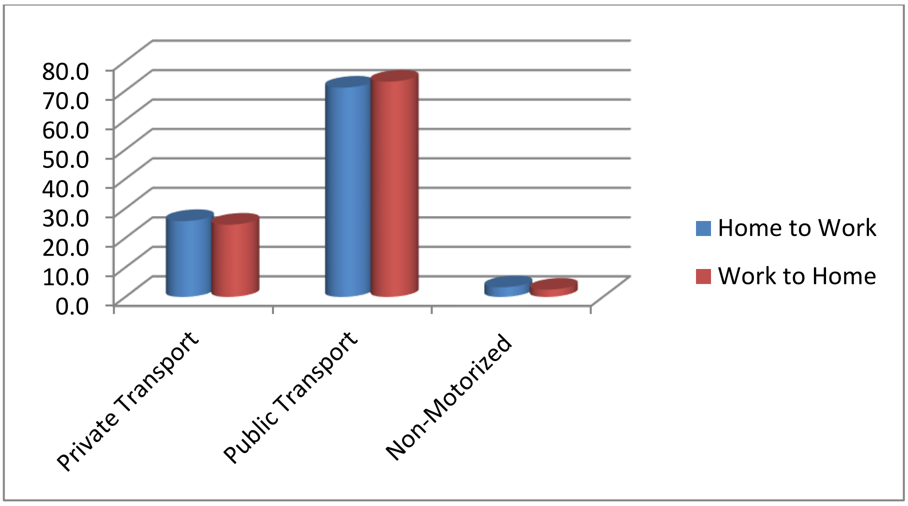


Figure 4.21- Commuting Mode Choice (Percentage)

4-3-4- Ideology

Employees' attitude toward telecommuting is very important and determines the level of employee's engagement in this pattern of working. In other words, ideology about different aspects of telecommuting acts as either hindrance or motivation.

Table 4.13 - Ideology of Employee on the Impact of Telecommuting

	Ideology about	Impact								Total	
		No		Low		Middle		High			
		Fre.	%	Fre.	%	Fre.	%	Fre.	%	Fre.	%
1	Family Psychic Health	12	4.9	21	8.6	104	42.4	108	44.1	245	100.0
2	Reducing Traffic Congestion	2	0.8	15	6.1	57	23.3	171	69.8	245	100.0
3	Work Efficiency	6	2.4	49	20.0	106	43.3	84	34.3	245	100.0

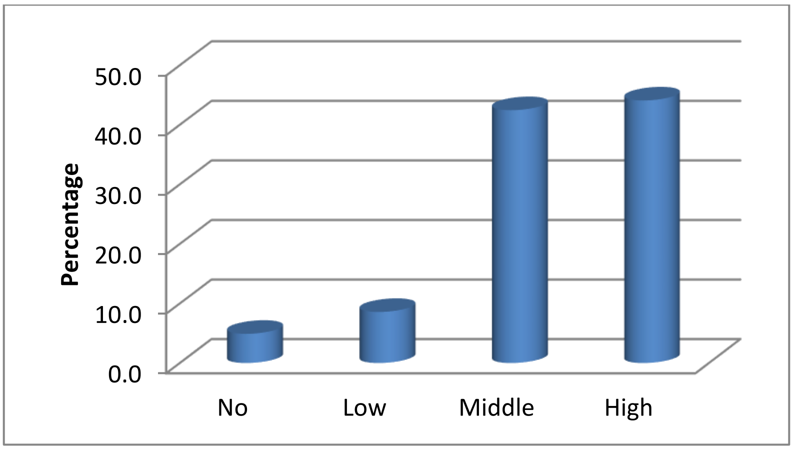


Figure 4.22 – Ideology of Employee on the Impact of Telecommuting on Family Psychic Health (Percentage)

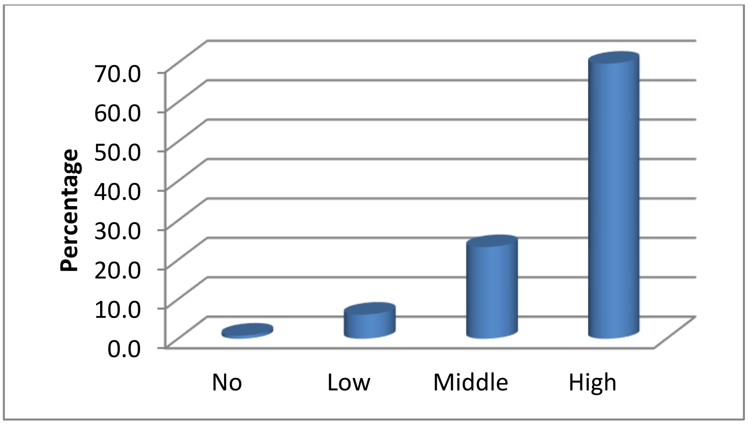


Figure 4.23 – Ideology of Employee on the Impact of Telecommuting on Reducing Traffic Congestion (Percentage)

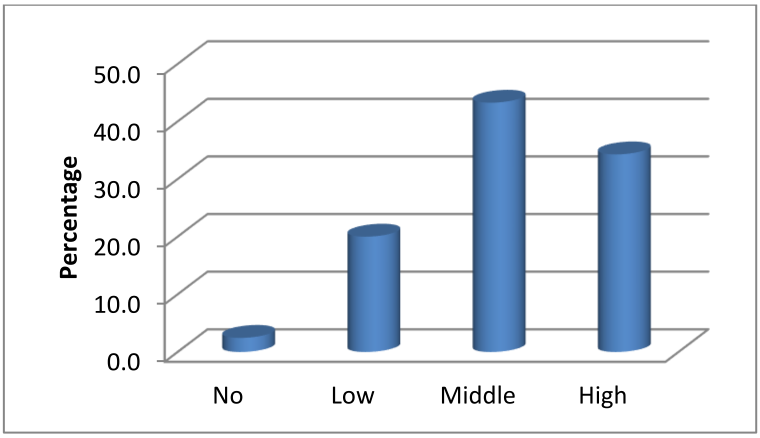


Figure 4.24 – Ideology of Employee on the Impact of Telecommuting on Work Efficiency (Percentage)

As can be seen from Figures 22-24, majority of employees have positive impression about telecommuting. Most of them have stated that adopting telecommuting would have good impact in their work quality, family wellbeing and reduces traffic congestion.

4-3-5- Telecommuting: Suitability and Preference

Suitability of telecommuting has been asked from both employees and supervisors in terms of number of days per week which suits employees. Table 4.14 and Figure 4.25 show that majority of supervisors mostly believe 1 or 2 days of telecommuting would be suitable for employees.

Table 4.14 - Suitable Days of Doing Telecommuting

Days	Employee		Supervisor	
	Fre.	%	Fre.	%
0	58	23.7	19	7.8
1	46	18.8	64	26.1
2	76	31	108	44.1
3	65	26.5	54	22
Total	245	100	245	100

The interesting difference is between 0 day of doing telecommuting as supervisors think most of the employees can do at least one day but around 24 percent of employees have stated that telecommuting is not suitable for them.

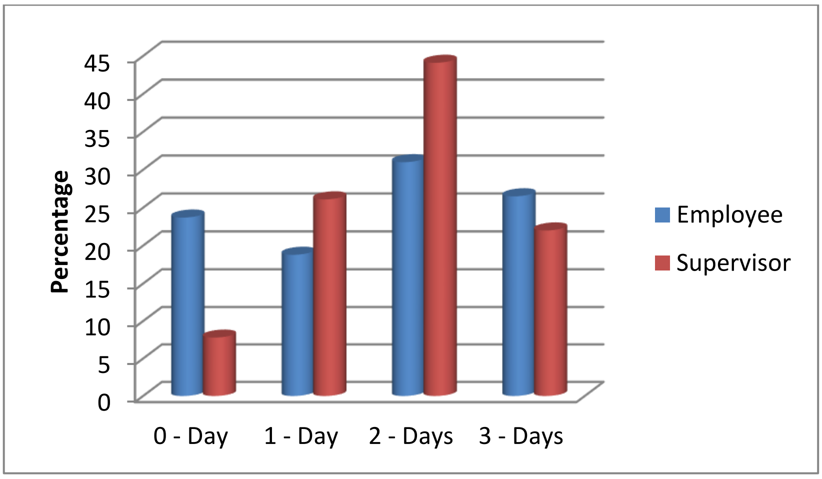


Figure 4.25 –Suitable Days of Doing Telecommuting (Percentage)

Apart from suitability, preference of employees towards telecommuting also has been asked in the questionnaire. As Table 4.15 and Figure 4.26 demonstrate, more than 4 days of telecommuting is not popular and most of the employees (around 50 percent) prefer to work 2 or 3 days from home per week.

Table 4.15 – Preference of Adopting Telecommuting

Days	Fre.	%
0	39	15.9
1	30	12.2
2	54	22.0
3	84	34.3
4	22	9.0
5	16	6.5
Total	245	100

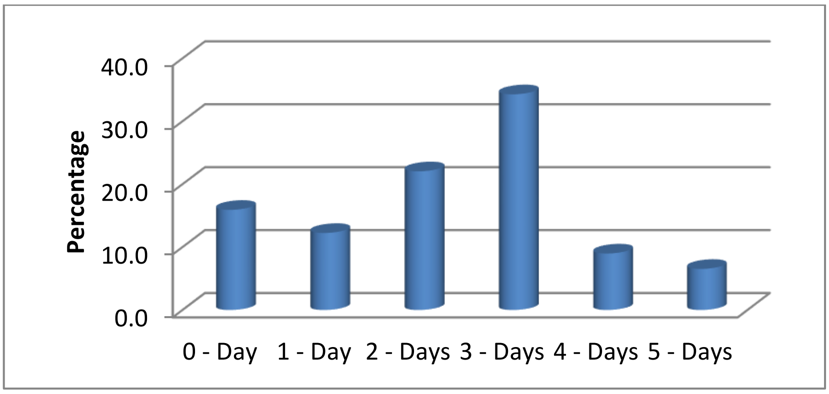


Figure 4.25 – Preference of Adopting Telecommuting (Percentage)

4-4-Conclusion

To study the suitability of telecommuting using the proposed approach in chapter 3, in the first step the available data set should be analysed. In this chapter, the gathered data from 7 organisations in central business district of Tehran, the capital of Iran, was analysed from different perspectives.

The data was gathered through a questionnaire consisting 26 questions. The sample size is 245 employees (70 women and 175 men). Job tasks were analysed using the concept of abstract job, which introduces job as composed of tasks rather than titles which are not very revealing. Also, sociodemographic factors, commuting variables, ideology and state preferences towards telecommuting were illustrated using different tables and graphs.

In the next chapter, the gathered analysed data is used to model suitability of telecommuting. The proposed novel approach for choice modelling using Fuzzy Networks (discussed in chapter 3) is tested and compared with the Random Utility Models using mentioned sample.

CHAPTER 5
MODELLING SUITABILITY OF TELECOMMUTING

5-1-Introduction

The main focus of this chapter is to model suitability of telecommuting using three different approaches and examine them via several performance indicators. As explained in chapters 2 and 3, Rule Based Network (RBN) as a novel approach is utilised in modelling telecommuting as a core of this study.

Adopting telecommuting is considered as a multidimensional decision making problem which various elements and factors have influence on adopting telecommuting. Better understanding of the internal decision process will help to develop more realistic and interpretable model. Based on various undertaken research, a proposed comprehensive internal decision process is converted to a transparent network and all external and internal variables and their relationships are identified.

The concept of choice models and maximising alternative utilities is used. To find utility of alternatives, RBN is tuned using Genetic Algorithm and fuzzy c-mean clustering approach. Dimensionality as an existing problem for RBN is tackled by input selection process. Not only significant external inputs are kept, but also less important intermediate variables and network branches are removed. Then, by linguistic composition techniques, the equivalent linguistic rules are obtained and maximised utilities are achieved. Based on the selected external inputs, Standard Fuzzy System (SFS) is developed to examine its performance against RBN. In addition, Multinomial Logit (MNL) model as a benchmark approach and discrete choice model for telecommuting is developed and comparison is made between these three models using different performance indicators for efficiency, transparency, accuracy and interpretability.

In this chapter, first a comprehensive conceptual framework and network for modelling telecommuting is discussed. Testing and training data sets are explained in Section 3. In

Section 4, a RBN model is developed and discussed in details for suitability of telecommuting. In Sections 4 and 5, SFS and MNL models are developed and in the last section, their performances are compared.

5-2-Telecommuting Conceptual Framework and Network

In chapter 2, few conceptual frameworks for modelling telecommuting were discussed. Among all of them, Mokhtarian and Salomon (1994) in their extensive study about telecommuting, developed a comprehensive conceptual model of the choice to telecommute. In their research and using travel behaviour theory, they tried to penetrate the black box of decision making procedure and proposed a model of internal decision making process (Figure 5.1).

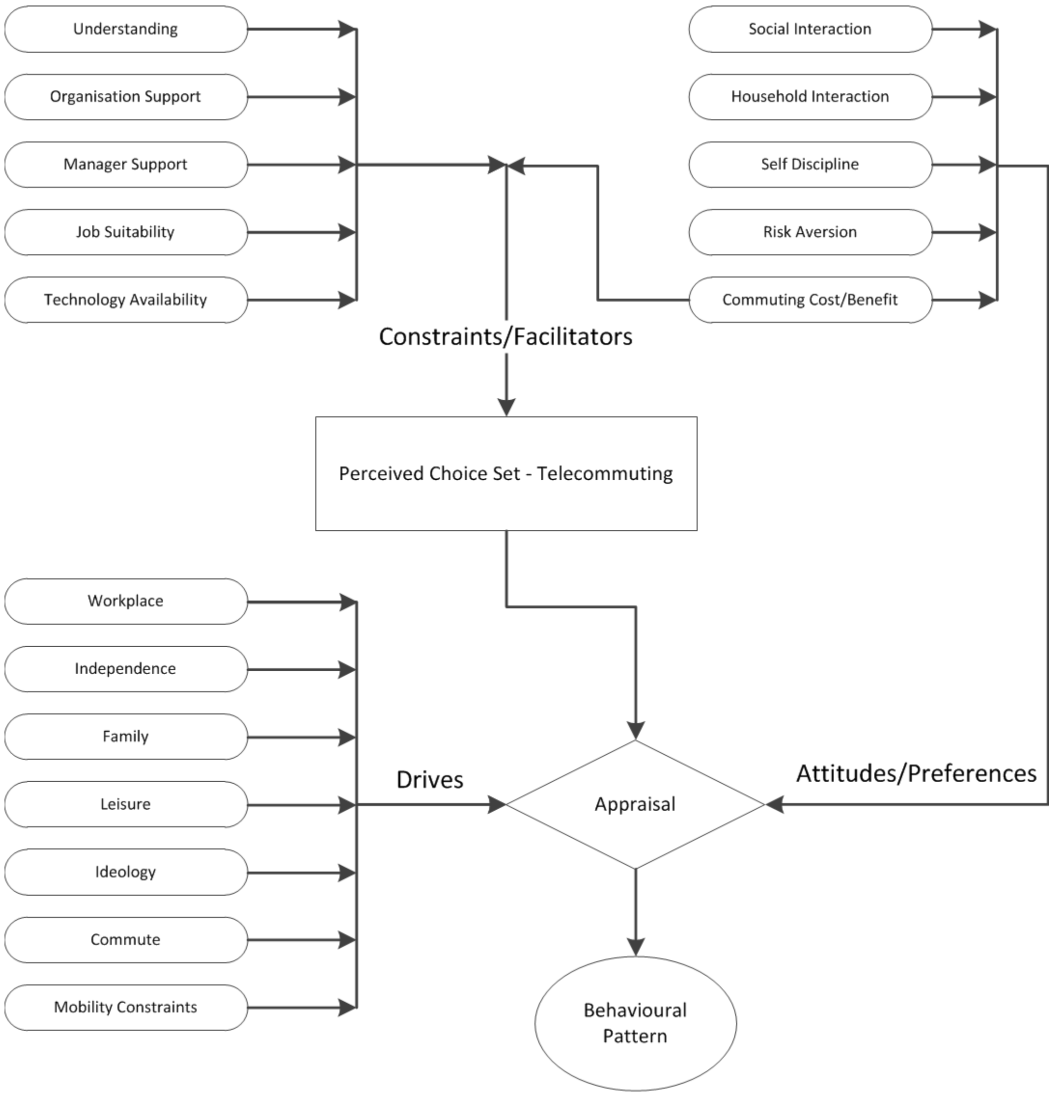


Figure 5.1: A schematic model of the internal decision making process for adopting telecommuting

As discussed before and Figure 5.1 shows above, 3 main components of the internal decision making process which are appraised by an employee for adopting telecommuting are drives, constraints/facilitators and preferences. Drive or motivator is a factor which actually motivates a person to consider a change to begin to telecommute. Constraint/facilitator factor may be either a constraint to prevent or hinders or allow changes for adopting telecommuting. A basic factor may be either constraint or facilitator depending on whether it is present in a positive sense or a negative one. Preference also shows the tendency to do telecommuting.

As adopting telecommuting is a multidimensional case, each component is derived from various categories of variables that describe that component. Mokhtarian and Salomon (1994) in their study identified different categories of each component (Figure 5.1) and also find relevant variables for categories. They also, collected data for their numerical studies based on this framework to develop a model for adopting telecommuting (Mokhtarian & Salomon, 1996a).

Although such a framework tries to get into the black box of internal decision making process and introduce a white-box and transparent framework which shows the intermediate variables between external inputs and final outputs, in modelling black-box approach has been considered for most of undertaken studies. In the other worlds using Multinomial Logit model all the inputs are fed to the model and outputs are obtained without considering the role of intermediate variables like drives, constraints and so on.

Rule based network helps for modelling suitability of telecommuting using fuzzy logic, as a powerful tool in decision making modelling, and also in a transparent approach to see the interaction between inputs, intermediates variables and outputs. In this way, the first step is to convert the established conceptual framework to form of a network that illustrates external inputs, intermediates variables, their interaction and finally their influence on final output.

Based on the shown proposed framework and also considering and localising the case of telecommuting, data was gathered in Tehran which explained statistically in chapter 4. Using that data set and by simplifying the framework, the network which can be used for rule based network is derived. In Figure 5.2, internal decision process has been modelled as a network which shows external inputs, intermediates variables, connection between them and also the output.

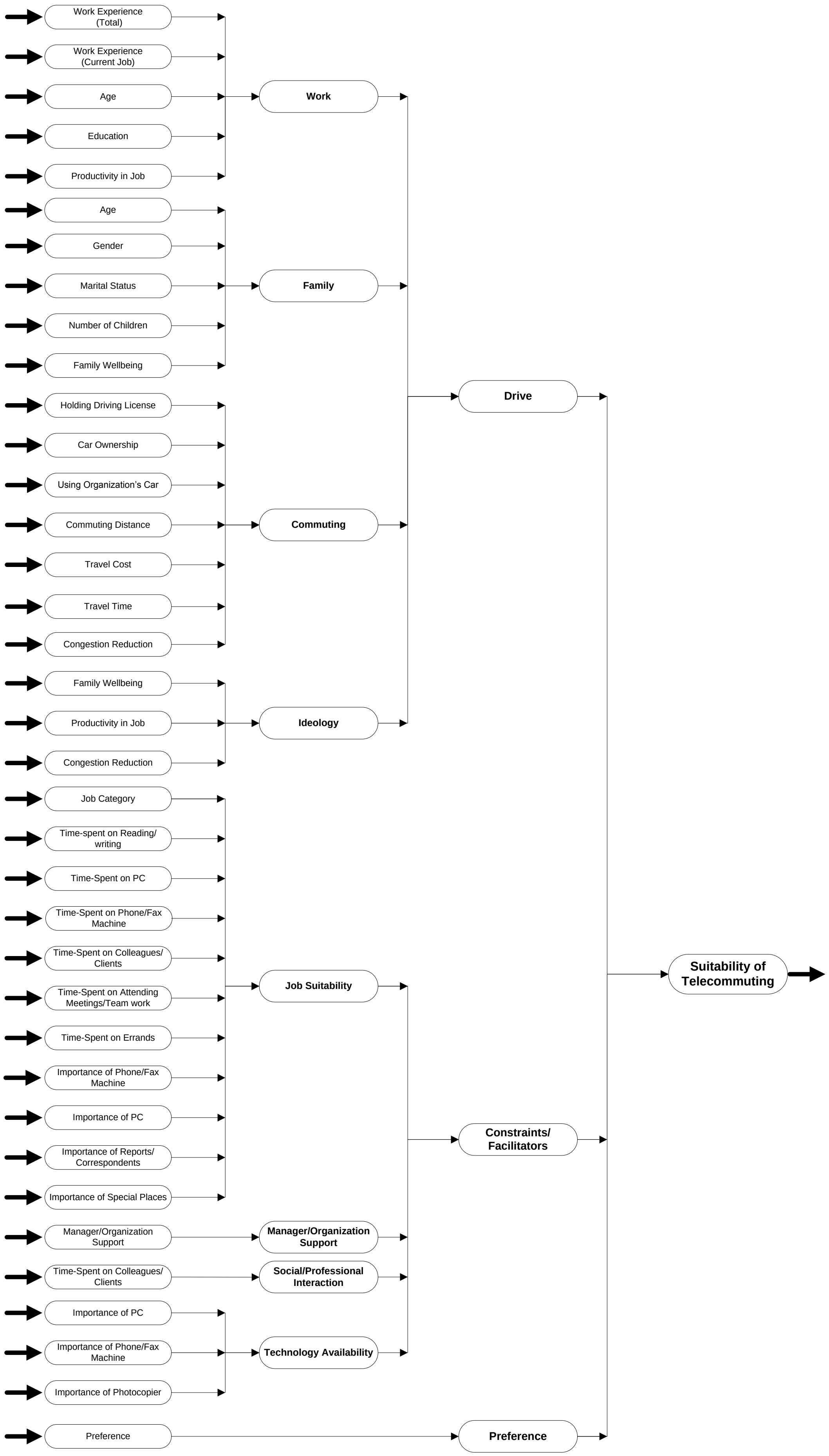


Figure 5.2: Internal decision making Network for modelling suitability telecommuting

In addition, the concept of abstract job (Mamdoohi et al., 2006), which is a way of considering jobs on the basis of their elements and tasks, is used for modelling suitability of the telecommuting. As a case in point, suitability of a job is assessed based on importance and the amount of time an employee spend on different tasks and devices rather than considering the title of jobs.

The derived network consists of 38 levels and 3 layers based on the existing conceptual framework and also available date set. It starts by the external inputs from far left to the output on the right side. Intermediate layers show the intermediate variables and connections are mapped by arrows. Such a structure well addresses Mokhtarian and Salomon (1997) concerns about composite variables. In their studies, factor analysis was used to reduce large sets of interrelated variables to smaller sets. But, here and using the concept and advantages of network, variables which have been recognised relevant by the conceptual framework are composed and generate intermediates variables. Having those variables, not only the influence of external inputs are seen on outputs, but also the role of each intermediates variables in describing output can be studied.

Three different models are developed for looking at suitability of telecommuting. In first approach, rule based network is used for finding utilities of alternatives and consequently probability of each choice is identified. In this method, the white-box and transparent network is used. In the other methods, using Standard Fuzzy System and multinomial logit models, as black-box approach, are developed and compared.

5-3-Training and Testing Data Sets

In order to develop models, selecting inputs and evaluating their performances, data is divided into training and testing data sets. Using training set, models are trained and also for RBN and MNL external inputs are selected. For model evaluation and seeing the performance of models, test set is used. In this study, sample data as explained in chapter 4 consists of 245 individuals and is divided randomly to 125 individuals for training set and to 120 individuals for test set.

5-4-Modelling Suitability of Telecommuting Using Fuzzy Network

First approach for modelling telecommuting is to use rule based network for getting utilities of different alternatives and by obtaining the utilities of all alternatives for individuals the choice probability of each alternative is achieved. To do so, it is needed to tune the model and select the most important inputs. Also, the role of intermediate variables should not be neglected as representatives of related variables that are fed with as well as determining the rules and membership functions.

In this study, there is no data for intermediate variables and only observations and external inputs are available. It is assumed that each intermediate variable carries the meaning and influence of its inputs on the final output and also next layer of intermediate variables. Thus, the values of each intermediate variable can be estimated as output of each sub-model. In the other words, each subsystem is calibrated using the observation and after selecting the most significant and important variables, the final values are calculated. Therefore, each intermediate variable has the taste of its inputs as merged variable and plays its role as medium between external inputs and the final output.

To do the horizontal and vertical merging for the whole network to get the utilities of alternatives, sub-systems should be tuned and important inputs should be selected. Since explained in chapter 3, MFs play the same role as coefficient do in normal regression models and for model calibration MFs should be optimised. So, for each subsystem, using the inputs and observation and Genfis3 function from MATLAB, rules (for simplicity 3 rules using fuzzy C-mean) are extracted and using Genetic Algorithm (GA), triangular (again for sake of simplicity) membership functions are optimised. As explained in chapter 3, the Gaussian MFs are obtained from Genfis3 function using fuzzy c-mean clustering methods and these MFs are approximated to triangular MFs for simplicity. It is also considered as initial solution as a chromosome in initial population to help GA for more efficient convergence. In the next step, important variables are selected using backward selection approach based on tuned model from the previous step. This approach is propagated through whole network to tune all the subsystems and eventually whole network and also select important external inputs and intermediate variables. The size of network will be decreased by cutting down non-important variables and intermediate variables and eventually branches of network.

Genetic Algorithm tries to minimise the difference between observation and model output by optimising the membership functions. An easy and possible solution for GA is to consider short-based triangles (Figure 5.3), which occupy a small piece of data range, and consequently membership values will be 0 and helps to get lower values for objective function. To resolve such a problem, some rules and restrictions are applied for GA to optimised membership functions in form of Figure 5.4. Having those restrictions, optimised membership functions have appropriate and meaningful overlap and cover the inputs and outputs domains. Also, not to have GA bias convergence based on specific chromosomes, GA is run 5 times for each subsystem and the best performance (optimised MFs) is selected.

Figure 5.3: Schematic membership functions which minimise the GA objective function without applying restrictions

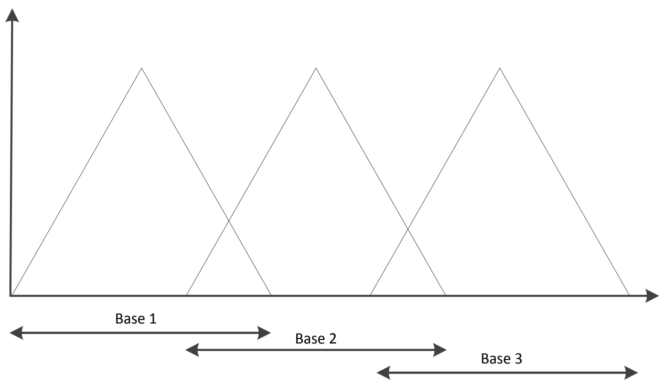


Figure 5.4: Schematic membership functions which minimise the GA objective function by applying restrictions

There are four alternatives (number of days per week) which have been chosen by employees using 0, 1, 2 and 3 in questionnaire. For simplicity and computational comfort, the range between 0 to 100 is considered for output and values 5, 35, 65 and 95 days are representatives respectively. Also, as there is no value for intermediate variables and should be obtained from subsystems, this range is considered for consistency.

5-4-1-Optimising Nodes in Fuzzy Network

In the rest of this section, each node as a sub-system (Figure 5.2) is examined and tuned as explained above:

5-4-1-1-Work-related drives

As Figure 5.5 shows, work related drives as an intermediate variable is obtained from 5 external inputs: total years of work experience, years of experience in current job, age, education and ideology about telecommuting in productivity in job. By merging these variables, the work related drive is made which shows the role of work as a concept in suitability of telecommuting. All variable have been discussed in chapter 4.

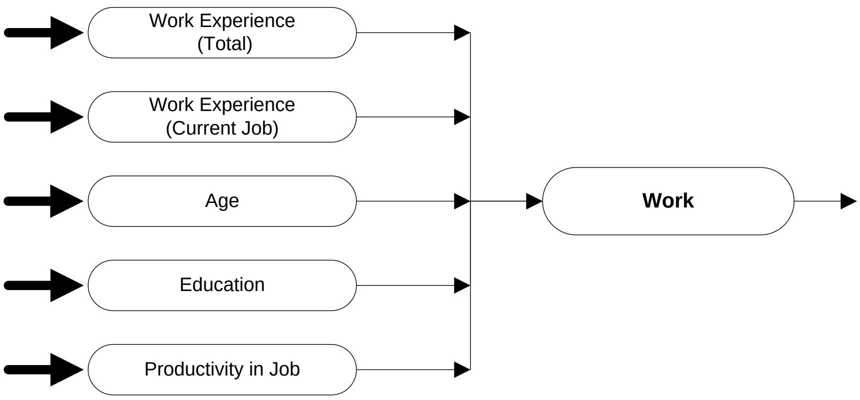
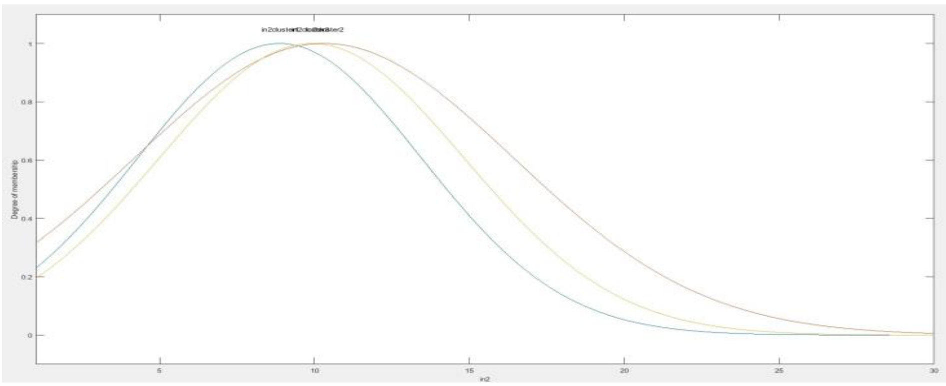


Figure 5.5: Work related drive as an intermediate variable

All 5 inputs and the observation from training data set are fed into Genfis3 MATLAB function and following Gaussian membership functions and rules are obtained. (Figure 5.6)



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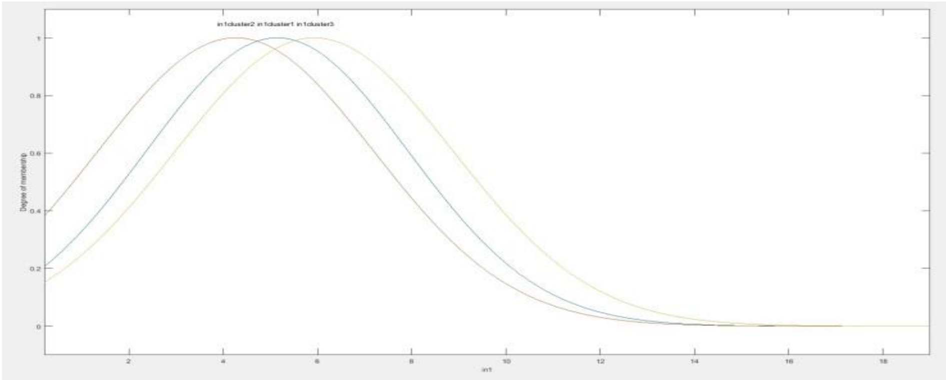

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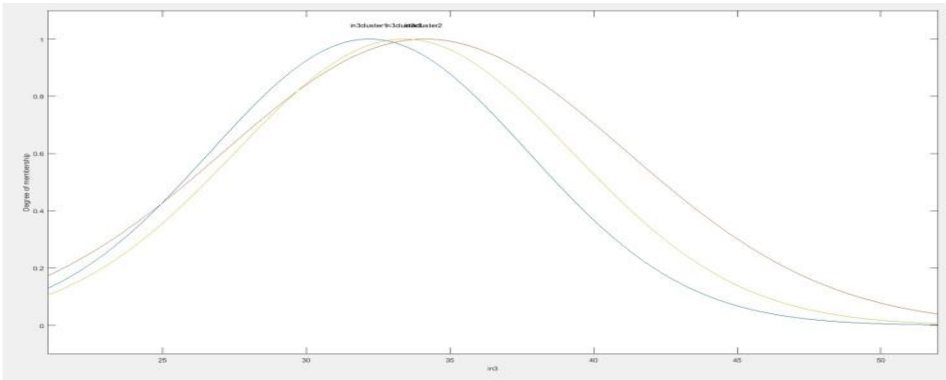

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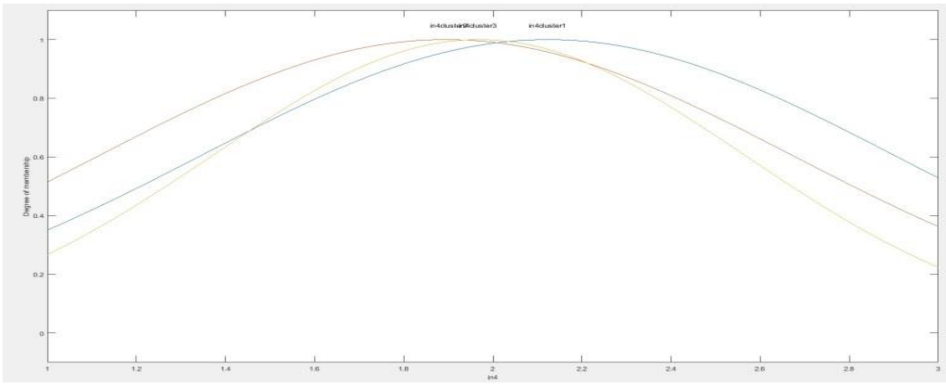

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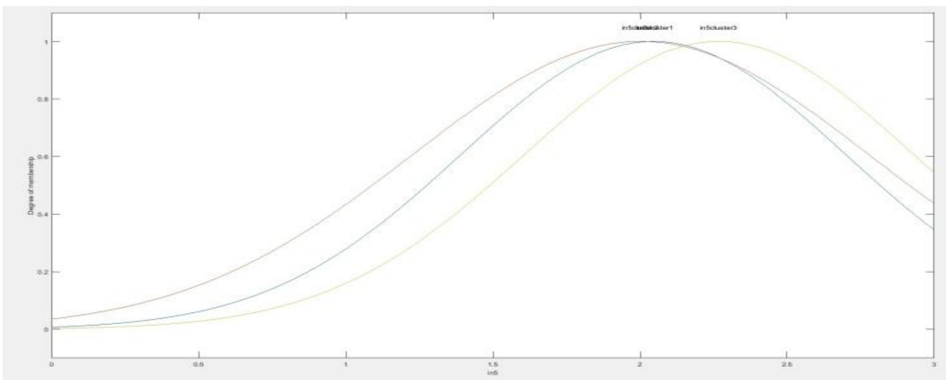


e)

Figure 5.6: Membership functions given by Genfis3 for work related drive variables:
a) Work Experience (total) b) Work Experience (current job) c) Age d) Education e) Productivity in Job

Also, based on fuzzy c-mean, 3 rules are obtained as follows:

Rule 1: If x_1 *is* A_{11} *and … and* x_m *is* A_{m1}*, then* y_1 *is* B_1

Rule 2: If x_1 *is* A_{12} *and … and* x_m *is* A_{m2}*, then* y_1 *is* B_2

Rule 3: If x_1 *is* A_{13} *and … and* x_m *is* A_{m3}*, then* y_1 *is* B_3

Which can be shown in format of the following matrix:

$$\begin{bmatrix} A_{11} & A_{21} & \dots & \dots & \dots & B_1 \\ A_{12} & A_{21} & \dots & \dots & \dots & B_2 \\ A_{13} & A_{31} & \dots & \dots & \dots & B_3 \end{bmatrix} \qquad (1)$$

For intermediate work-related drive variable the following 3 rules are achieved from Gengfis3 function:

1) If *Work Experience (total)* is *low* and *Work Experience (current job)* is *middle* and *Age* is *low* and *Education* is *high* and *Productivity in Job* is *middle* then *Work Related Drive* is *high*
2) If *Work Experience (total)* is *high* and *Work Experience (current job)* is *low* and *Age* is *high* and *Education* is *low* and *Productivity in Job* is *low* then *Work Related Drive* is *low*
3) If *Work Experience (total)* is *middle* and *Work Experience (current job)* is *high* and *Age* is *middle* and *Education* is *middle* and *Productivity in Job* is *high* then *Work Related Drive* is *middle*

Rules matrix can be presented as shown in (1) when low, middle and high are represented as 1, 2 and 3 respectively. First five columns show the antecedents and the last column is consequent.

$$\begin{bmatrix} 1 & 2 & 1 & 3 & 2 & 3 \\ 3 & 1 & 3 & 1 & 1 & 1 \\ 2 & 3 & 2 & 2 & 3 & 2 \end{bmatrix} \qquad (2)$$

Using the above rules and triangular approximation of membership functions (as shown in Figure 3.8) and also genetic algorithm, the subsystem is tuned and triangular membership functions are optimised. As mentioned in methodology chapter, GA stops after 1000 iterations or converging to a minimised valued for 50 times. Figure 5.7 shows how the GA minimises the objective function which is to make the observation close to the model output by optimising MFs (Figure 5.8).

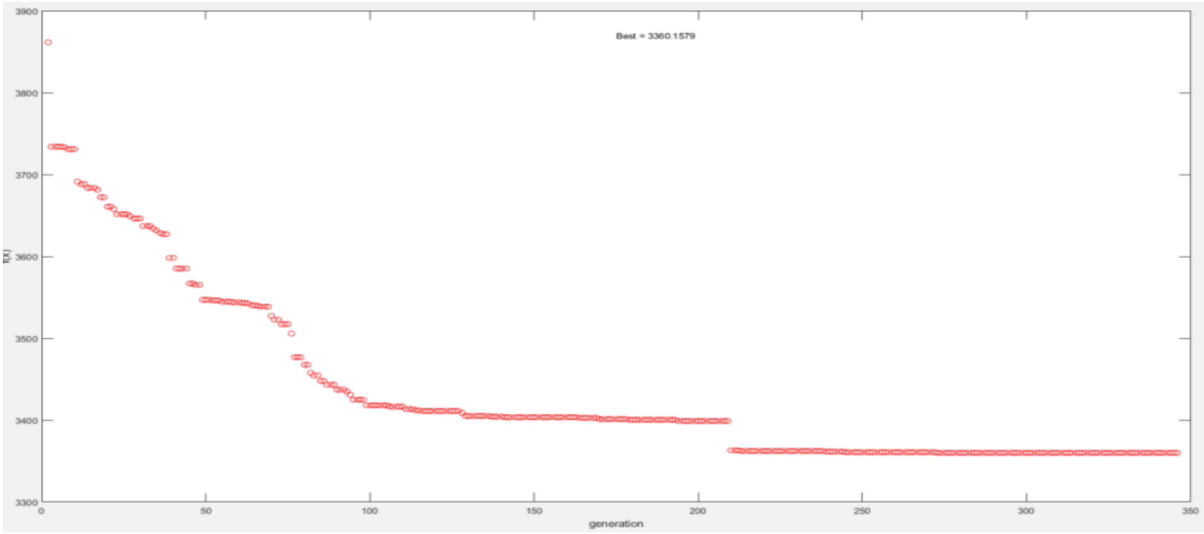
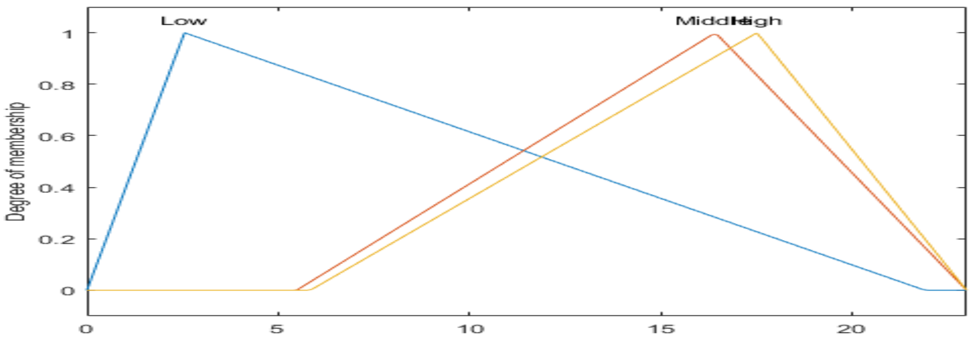


Figure 5.7: GA minimisation for subsystem work related drive



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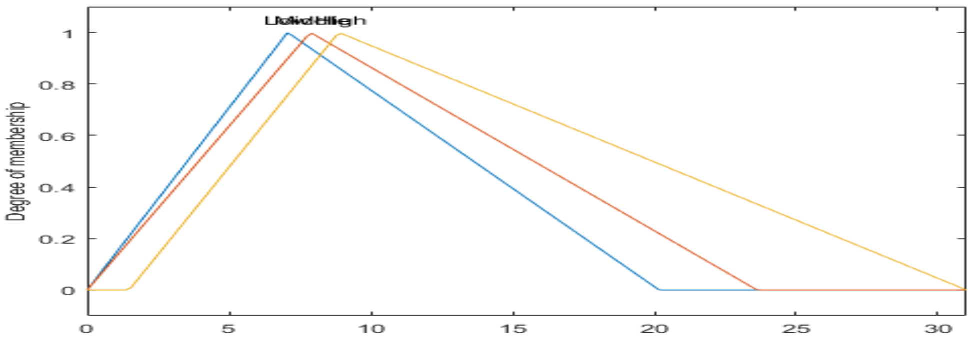

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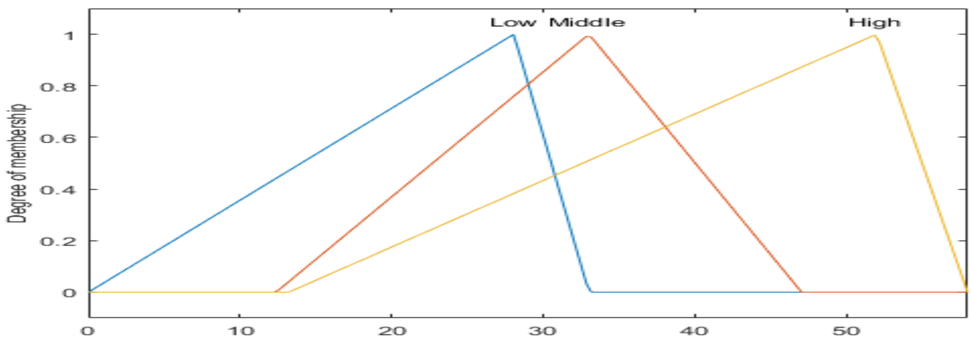

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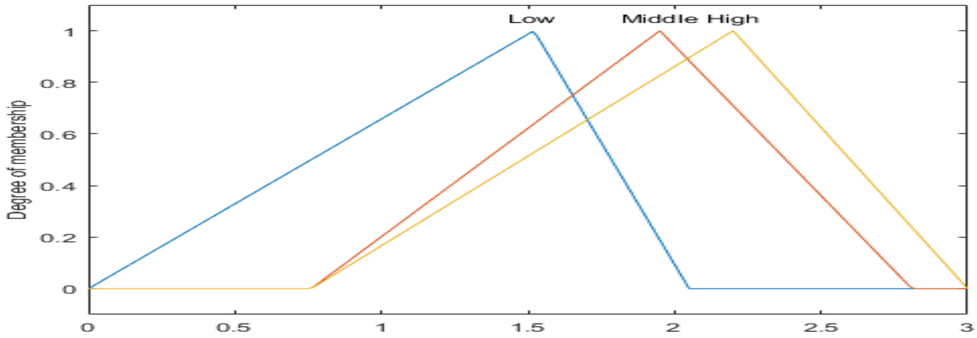

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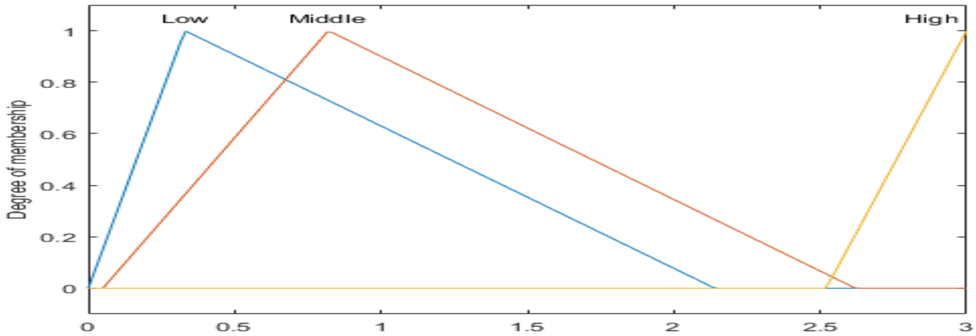

e)

Figure 5.8: Optimised membership functions for work related drive

a) Work Experience (total) b) Work Experience (current job) c) Age d) Education e) Productivity in Job

The next step is to select the most important variables using the backward selection method explained in chapter 3 using the test data set. First the tuned model is fed by the test data set and the error term is calculated (J_c criterion (Chiu, 1996)), then 1 variable is removed from the model and the best model with lowest J_c is selected. By this approach variables one by one are removed from the initial tuned model till no more variable remains (Figure 5.9). As can be seen from Figure 5.9, to have error minimised error or least value for J_c criterion, three variables should be kept in model: *Work experience (current job)*, *Education* and *Ideology of Productivity in Job*.

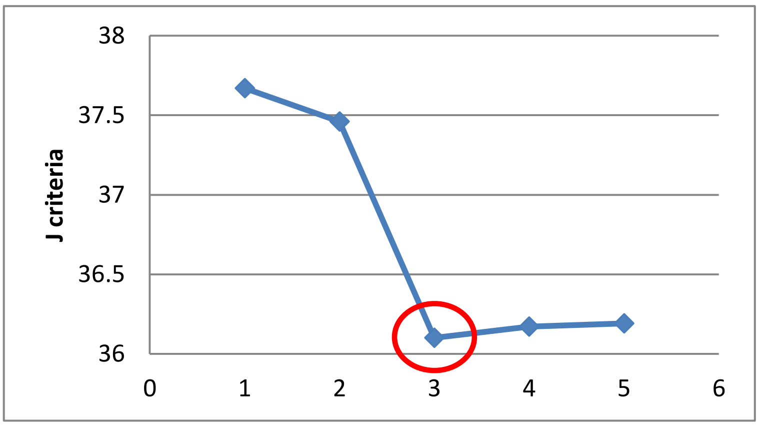

Figure 5.9: Number of input variables remaining for work related drive

5-4-1-2-Family-related drives

The next subsystem is family related drives and its output or the intermediate variable here shows the influence of related external inputs on suitability of telecommuting from family point of view (Figure 5.10).

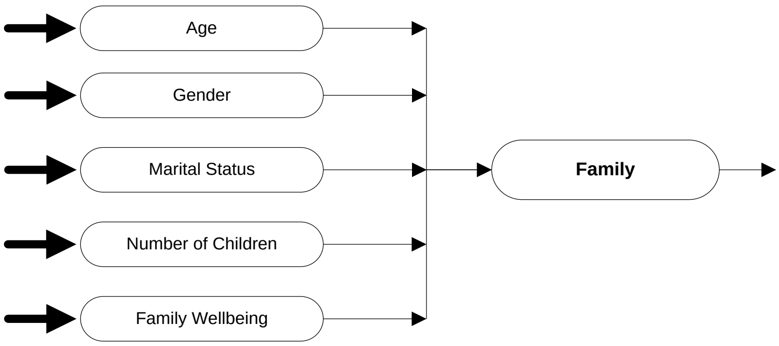

Figure 5.10: Family related drive as an intermediate variable

Using Genfis3 function from MATLAB, the initial membership functions as well as the rules are extracted from the data shown in Figure A1.1 (Appendix 1). Also rules are drawn from the data using MATLAB as:

1) If *Age* is *high* and *Gender* is *high* and *Marital Status* is *middle* and *Number of Children* is *high* and *Family wellbeing* is *low* then *Family Related Drive* is *low*
2) If *Age* is *middle* and *Gender* is *middle* and *Marital Status* is *high* and *Number of Children* is *middle* and *Family wellbeing* is *middle* then *Family Related Drive* is *middle*
3) If *Age* is *low* and *Gender* is *low* and *Marital Status* is *low* and *Number of Children* is *low* and *Family wellbeing* is *high* then *Family Related Drive* is *high*

Rules matrix for this subsystem is shown below:

$$\begin{bmatrix} 3 & 3 & 2 & 3 & 1 & 1 \\ 2 & 2 & 3 & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 & 3 & 3 \end{bmatrix} \tag{3}$$

Worth noting that for variables like gender or marital status, linguistic variables such as low or medium seem to be meaningless, but rules are drawn from the data not expert knowledge. It can be explained as only labels and just adjust the influence of that variable in model. Also, fuzziness has broader range and meaning and the above mentioned issue can be justified in light of fuzzy logic.

Having the rules and initial membership function (approximation) are fed into GA model and optimised MFs are obtained. Figure A2.1 (Appendix 2) shows the minimisation process of GA versus number of generations and iterations. As can be seen, after 260 iterations, minimum value is obtained for objective function. Consequently, optimised membership functions are obtained from GA minimisation shown in Figure A3.1 (Appendix 3).

Using the test data set, backward selection method and removing variable one by one from the model the following Figure is obtained (Figure 5.11).

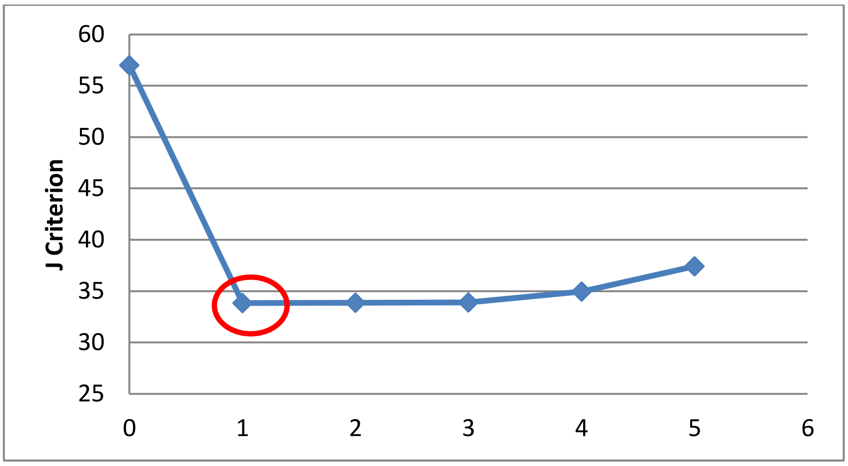

Figure 5.11: Number of input variables remaining for family related drive variables

As can be seen from Figure 5.14, only one variable can be selected for family related drives subsystem: *Marital Status*

5-4-1-3-Commuting-related drives

The next subsystem is commuting related drives and the influences of all relevant external inputs are seen on that intermediate variable (Figure 5.12). At the first step, using Genfis3 function of MATLAB, all the rules and initial MFs are extracted (Figure A1.2 – Appendix 2)

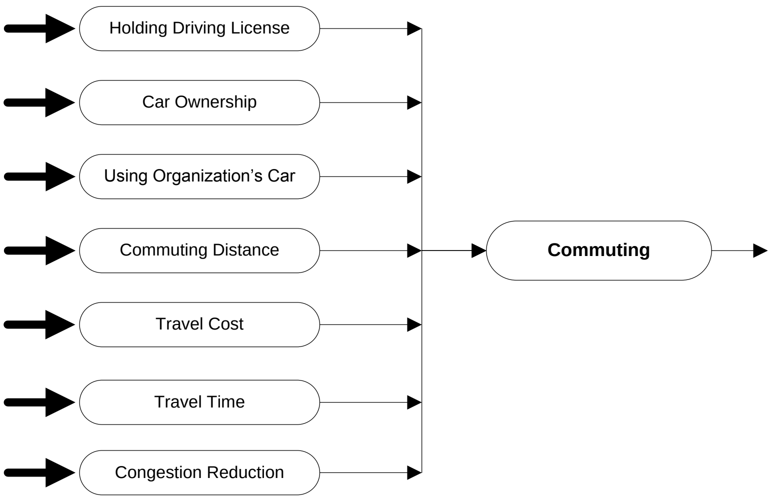

Figure 5.12: Commuting related drives as an intermediate variable

And the rules are:

1) If *Holding driving license* is *high* and *Car ownership* is *low* and *Using organisation's car* is *high* and *Commuting distance* is *low* and *Travel cost* is *low* and *Travel time* is *low* and *Ideology on congestion reduction* is *low* then *commuting related drive* is *low*
2) If *Holding driving license* is *middle* and *Car ownership* is *middle* and *Using organisation's car* is *middle* and *Commuting distance* is *high* and *Travel cost* is *middle* and *Travel time* is *middle* and *Ideology on congestion reduction* is *middle* then *commuting related drive* is *high*
3) If *Holding driving license* is *low* and *Car ownership* is *high* and *Using organisation's car* is *low* and *Commuting distance* is *middle* and *Travel cost* is *high* and *Travel time* is *high* and *Ideology on congestion reduction* is *high* then *commuting related drive* is *middle*

Rules can be written as a matrix below:

$$\begin{bmatrix} 3 & 1 & 3 & 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 3 & 2 & 2 & 2 & 3 \\ 1 & 3 & 1 & 2 & 3 & 3 & 3 & 2 \end{bmatrix} \tag{4}$$

The subsystem is tuned using the above rules and initial MFs obtained from Genfis3 using GA. Figure A2.2 (Appendix 2) shows how GA minimises the objective function and optimised the membership functions. Also, Figure A3.2 (Appendix 3) depicts the optimised MFs as GA output. The tuned model is used for feature selection using backward selection method.

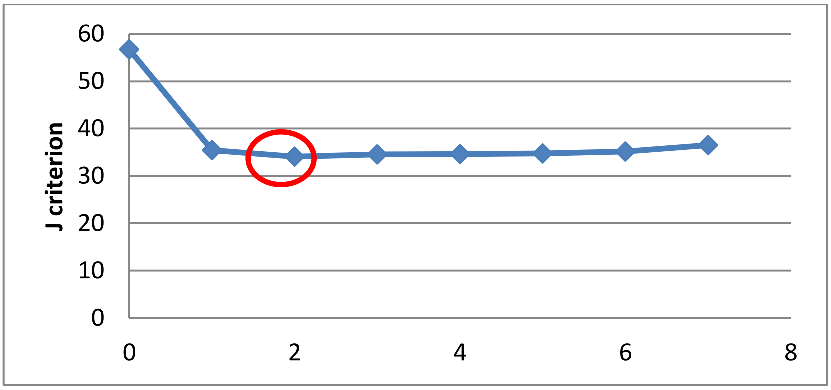

Figure 5.13: Number of input variables remaining for Commuting related drives variables

As Figure 5.13 shows, the minimum error happens when two variables *Car ownership* and *Commuting distance* are selected for commuting related variables.

5-4-1-4-Ideology-related drives

Ideology related drive variable is composed of three variables which are the employee's ideology about the impact of telecommuting in family wellbeing, productivity in job and traffic congestion reduction (Figure 5.14).

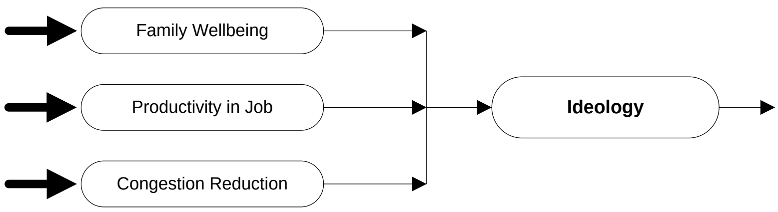

Figure 5.14: Ideology related drives as an intermediate variable

Same as explained above, initial membership functions and rules are derived from Genfis3 function (Figure A1.3 - Appendix 1). Also, the extracted rules are shown below:

1) If *Family wellbeing* is *low* and *Productivity in job* is *low* and *Congestion reduction* is *low* then *Ideology Related Drive* is *low*
2) If *Family wellbeing* is *middle* and *Productivity in job* is *middle* and *Congestion reduction* is *high* then *Ideology Related Drive* is *middle*
3) If *Family wellbeing* is *high* and *Productivity in job* is *high* and *Congestion reduction* is *middle* then *Ideology Related Drive* is *high*

Rule matrix can also be expressed as follows:

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 2 & 3 & 2 \\ 3 & 3 & 2 & 3 \end{bmatrix} \quad (5)$$

The training data set (125 individuals data) and above extracted rules as well as approximation of initial MFs obtained from Genfis3 are used to optimise the MFs. Genetic algorithm iterations and minimisation approach is shown in Figure A2.3 (Appendix 2) and minimised value for objective function is achieved after more than 170 iterations.

The output of GA is set of membership functions that minimise the objective function which tries to minimise the distance between observation and model output. Figure A3.3 (Appendix 3) shows the optimised MFs for Ideology drive related variables

Backward selection approach, optimised MFs, extracted rules using fuzzy c-mean and the test data set are all used to find the most important variables. As it can be seen from Figure 5.15 and using J_c criterion, only one the variables plays the most important role in this subsystem which is *Ideology on traffic reduction*.

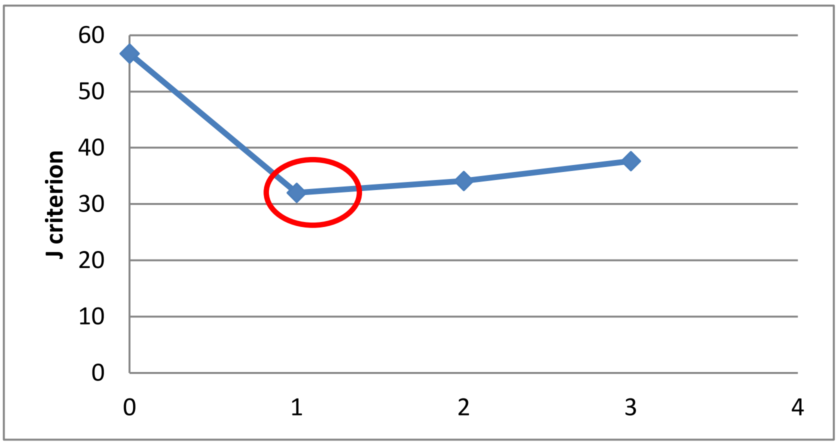

Figure 5.15: Number of input variables remaining for Ideology related drives variables

5-4-1-5-Job suitability-related Constraints/Facilitators

Another factor which plays a crucial role in adopting telecommuting is the suitability of the job. Job can be either constraints or facilitator for adopting telecommuting, so suitability of employee's job is considered in this category. In this study, the concept of abstract job (Mamdoohi et al., 2006)is used. In this approach, instead of picking only the job titles, 11 tasks and components of jobs are considered for suitability of adopting telecommuting (Figure 16). The assumption is that employees think about the suitability of their job by considering all these factors together. Therefore, the intermediate variable of job suitability, which is made of these components, is the representative of all those factors. This variable can be used for seeing the influence of job suitability on the constraints/facilitators intermediate variables and also output.

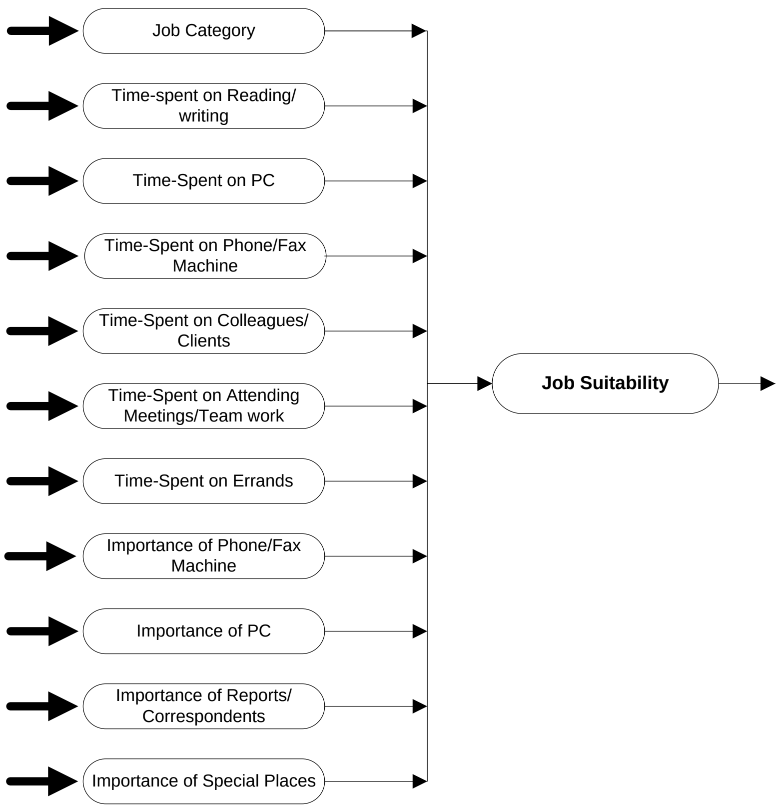

Figure 5.16: Job suitability-related Constraints/Facilitators as an intermediate variable

Genfis3 function of MATLAB is used to extract all the if-then rules and also the primary Gaussian MFs as initial solution for tuning the subsystem. Figure A1.4 (Appendix 1) shows all the MFs obtained from Genfis3.

For the job suitability intermediate variables, 3 rules are extracted. As the antecedents and consequent of these if-then rules have 10 parts, for sake of simplicity only the matrix of rules are presented below (5). First 9 columns are antecedents and 1, 2 and 3 are the representative of low, middle and high linguistic variables respectively. Also, 10^{th} column is the consequent or then part of the rules.

$$\begin{bmatrix} 1 & 1 & 3 & 3 & 2 & 1 & 1 & 1 & 3 & 1 & 2 & 3 \\ 2 & 3 & 2 & 1 & 1 & 3 & 2 & 2 & 2 & 3 & 1 & 2 \\ 3 & 2 & 1 & 2 & 3 & 2 & 3 & 3 & 1 & 2 & 3 & 1 \end{bmatrix} \tag{6}$$

Above rules as well as triangular approximation of the given MFs are used to tune the subsystem using the training data set. GA after around 300 iterations gives the optimised MFs as shows in the Figure A2.4 (Appendix 2). The optimised MFs are presented in Figure A3.4 (Appendix 3).

Having the job suitability subsystem tuned, important inputs can be selected. Similarly to other intermediate variables, backwards selection method is implemented for input selection. The test data set is used and as can be seen from the Figure 5.17, 4 inputs are chosen as important external variables for this node for presenting suitability of job regarding adopting telecommuting: *Job categories*, *Time-spent on PC*, *Time-spent on Phone/Fax* and also *Importance of special places like lab* and etc.

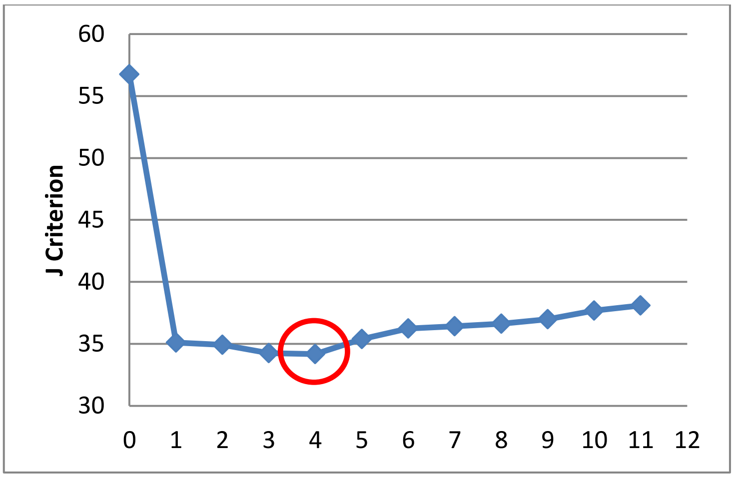

Figure 5.17: Number of input variables remaining for Job suitability-related related Constraints/Facilitators variables

5-4-1-6-Technology Availability-related Constraints/Facilitators

Next intermediate variable is technology availability. The availability of technology can also be either constraint or facilitator for adopting telecommuting. In case of having technological requirements around at home, working from home would be more suitable. This intermediate variable is seen in the model by considering 3 external inputs which are *Importance of PC, Phone/Fax Machine* and *Photocopier machine*. (Figure 5.18)

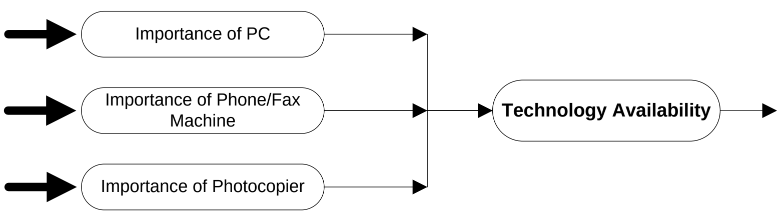

Figure 5. 18: Technology Availability-related Constraints/Facilitators as an intermediate variable

Similar to other discussed nodes, Genfis3 function is used for extracting rules and initial MFs (Figure A1.5 – Appendix 1). Rules and the matrix are shown as follows:

1) If *Importance of PC* is *low* and *Importance of Phone/Fax Machine* is *low* and *Importance of Photocopier* is *Middle* then *Technology Availability* is *low*
2) If *Importance of PC* is *Middle* and *Importance of Phone/Fax Machine* is *high* and *Importance of Photocopier* is *high* then *Technology Availability* is *high*
3) If *Importance of PC* is *high* and *Importance of Phone/Fax Machine* is *Middle* and *Importance of Photocopier* is *low* then *Technology Availability* is *Middle*

$$\begin{bmatrix} 1 & 1 & 2 & 1 \\ 2 & 3 & 3 & 3 \\ 3 & 2 & 1 & 2 \end{bmatrix} \qquad (7)$$

To get the optimised MFs, GA (Figure A2.5 – Appendix 2) uses the train data, above rules and also initial solution (MFs triangular approximations). The output of genetic algorithm is optimised MFs as shown in Figure A3.5 (Appendix 3). The tuned subsystem is used for input selection using backward selection method. As Figure 5.19 shows, only 1 input is important: *Importance of PC*.

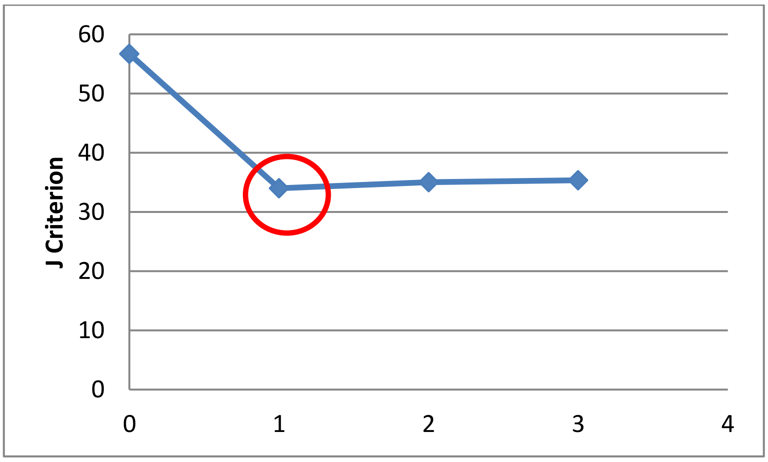

Figure 5.19: Number of input variables remaining for Technology Availability-related Constraints/Facilitators variables

5-4-1-7-Drives

In the next layer of the proposed network, 3 main factors (intermediate variables) have been shown which directly influence on the decision making process by the employees. The first one is drives, which represent the factors that motivate employees to adopt telecommuting. As Figure 5.2 shows, four intermediate variables make drive variable are work, family, commuting and ideology related drives variable.

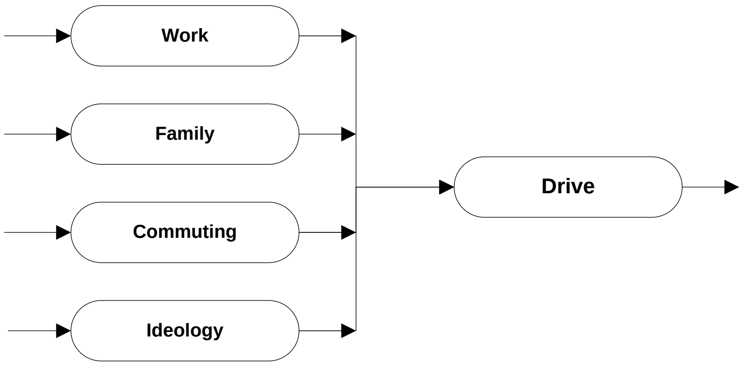

Figure 5.20: Drives as an intermediate variable

From the previous layer (layer 1) and using the tuned subsystems and selected inputs, values for the four inputs (work, family, commuting and ideology) are obtained. As explained earlier, the range of intermediate variables' are between 0 till 100 and can be considered individually. Therefore, various analyses can be undertaken and find the relationship and correlation between them and output and so on. In this study, the focus is to use their values for feed the next layer's intermediate variables.

Four intermediate variables are obtained using tuned with selected inputs and trained data. Having values for above mentioned four variables, drive node can be tuned. For the first step, initial MFs and rules are achieved using Genfis3 (Figure A1.6 – Appendix 1).

Rules are:

1) If *Work-related drive* is *low* and *Family-related drive* is *Middle* and *Commuting-related drives* is *Middle* and *Ideology-related drives* is *low* then *Drives* is *low*
2) If *Work-related drive* is *Middle* and *Family-related drive* is *low* and *Commuting-related drives* is *low* then *Ideology-related drives* is *high* then *Drives* is *high*
3) If *Work-related drive* is *high* and *Family-related drive* is *high* and *Commuting-related drives* is *high* then *Ideology-related drives* is *Middle* then *Drives* is *Middle*

Rule matrix can also be expressed as follows:

$$\begin{bmatrix} 1 & 2 & 2 & 1 & 1 \\ 2 & 1 & 1 & 3 & 3 \\ 3 & 3 & 3 & 2 & 2 \end{bmatrix} \quad (8)$$

Drive subsystem is tuned using GA to find the optimum MFS. Figure A2.6 (Appendix 2) shows the minimisation process and Figure A3.6 (Appendix 3) illustrates the optimised MFs.

For the drives intermediate variable, important inputs should be selected. Inputs for the intermediate variable in the second level are the output of the previous layer intermediate variables. In other words, by selecting inputs for this second layer or removing inputs or connections, a branch of the network will be removed. Actually, input selection at this stage means selecting important branches of network that are fed to that intermediate variable.

For the drives intermediate variable, input selection is done using the backward selection method as explained before. Test data is used for selecting the inputs. As there is no value for test data set for intermediate variables, tuned model is used to produce the values for connecting variables. Since Figure 5.21 shows, two important variables should be selected which are *Work and Family- related drives*.

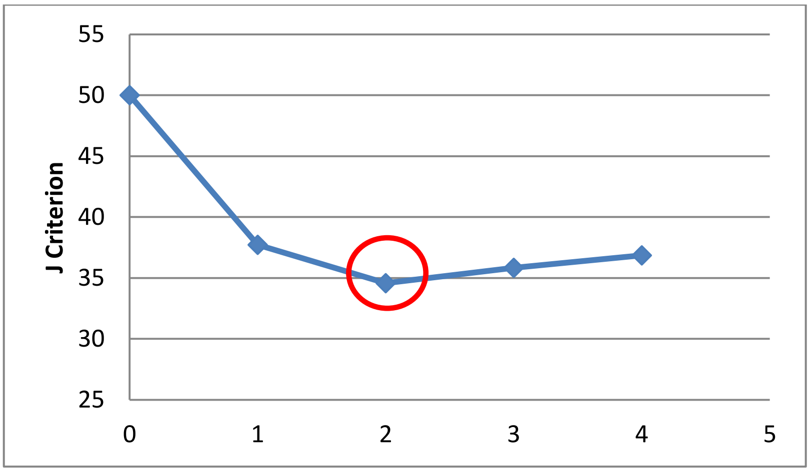

Figure 5.21: Number of input variables remaining for Drives variables

It simply implies the role of family and work in adopting telecommuting. In other words and based on selected inputs for family and work related drives intermediate variables, marital status, job experience in current job, education and thinking of telecommuting as an productive approach in work are the main stimulators for adopting telecommuting.

5-4-1-8-Constraints/Facilitators

Inputs for constraints/facilitators are Job suitability, Manager/Organisation support, Social/professional interaction and Availability of technology as Figure 5.22 shows.

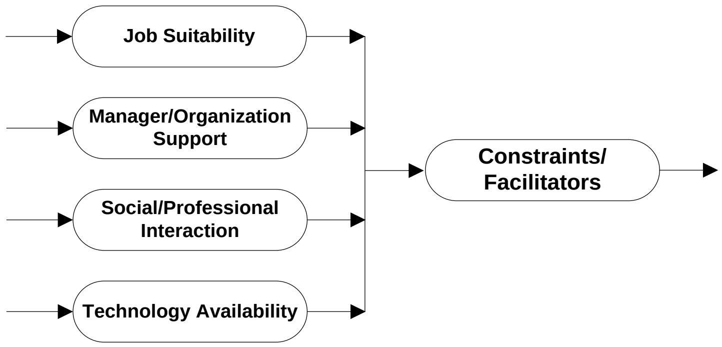

Figure 5.22: Constraints/Facilitators as an intermediate variable

As explained for Drives intermediate variable which is in the same layer as Constraints/Facilitators is, feeding variables are obtained from the intermediate variables in the previous layer. To get the initial solution for the GA, Genfis3 is used for MFs and rules matrix. Figure A1.7 (Appendix 1) shows the initial MFs.

Obtained rules are:

1) If *Suitability of job* is *low* and *Managerial/Organisational Support* is *low* and *Social/Professional Interaction* is *high* and *Availability of technology* is *low* then *Constraints/Facilitator* is *low* (constraint)
2) If *Suitability of job* is *Middle* and *Managerial/Organisational Support* is *Middle* and *Social/Professional Interaction* is *low* and *Availability of technology* is *Middle* then *Constraints/Facilitator* is *Middle* (so so)
3) If *Suitability of job* is *high* and *Managerial/Organisational Support* is *high* and *Social/Professional Interaction* is *Middle* and *Availability of technology* is *high* then *Constraints/Facilitator* is *high* (facilitator)

Rule matrix can also be expressed as follows:

$$\begin{bmatrix} 1 & 1 & 3 & 1 & 1 \\ 2 & 2 & 1 & 2 & 2 \\ 3 & 3 & 2 & 3 & 3 \end{bmatrix} \tag{9}$$

Using initial MFs, rules, trained data and GA are used (figure A2.7 – Appendix 2) for finding the optimised MFs (Figure A3.7 – Appendix 3) for feeding variables.

Important branches are selecting as explained earlier and Figure 5.23 shows. *Job Suitability* and *Technology Availability* play the most important rule in defining Constraints/Facilitators intermediate variable.

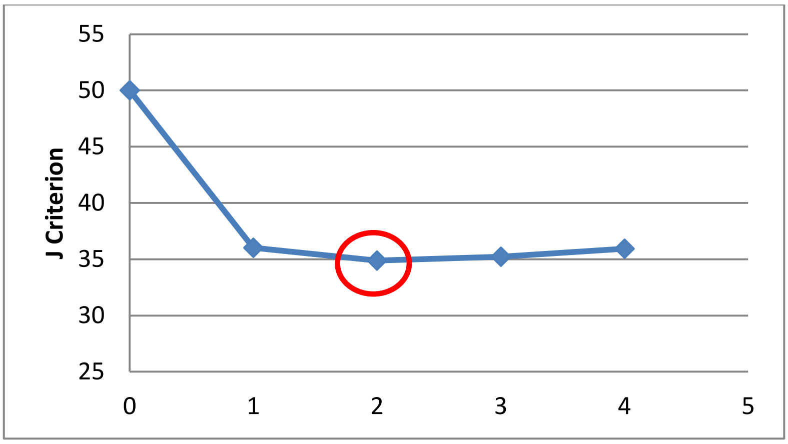

Figure 5.23: Number of input variables remaining for Constraints/Facilitators variables

5-4-1-9-Suitability of Telecommuting

The last node as Figure 2 shows is the suitability of telecommuting or final output. The main issues that play the crucial role in making decision about adopting telecommuting are Drives, Constraints/Facilitators and Preference. The first two factors have been obtained from the previous layers and only preference is considered as external input to this node (Figure 5.24).

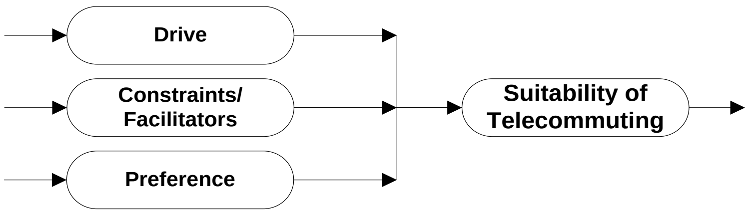

Figure 5.24: Suitability of Telecommuting as final output

In this case study, 4 alternatives exist for employees. As mentioned at the beginning of this section, alternatives 0, 1, 2 and 3 days of doing telecommuting converted to 5, 35, 65 and 95 respectively for computational purposes in the range between 0 to 100 (to be compatible with intermediate variables' range). In the last layer of model, to be able to see the linguistic variables for each alternative as an output and also to be in line with the initial assumption, some modifications are undertaken. For alternative 0 days, observation values 5, 35, 65 and 95 are replaced by 95, 65, 35 and 5 respectively to show the highest suitability for 0 alternative which initially represented by 5. Likewise for alternative 1, those initial values are converted to 35, 95, 65 and 5 to emphasise that alternative 1 has the highest suitability. The same for alternative 2 and 3, these sets will be 5, 35, 95, 65 and 5, 35, 65 and 95 respectively.

5-4-1-9-1-Suitability of Telecommuting for 0 Day

For looking at suitability of telecommuting for 0 days, the observations values are 95, 65, 35 and 5 (for alternative 0,1,2 and 3) which means the highest value is allocated to 0 days and lowest to 3 days of doing telecommuting. Using such data for observation and getting the values for input variables to these nodes (intermediate variables) from last layer, the last bit of the FN can be tuned. Figure A1.8 (Appendix 1) shows the initial MFs using Genfis3.

Obtained rules are:

1) If *Drives* is *low* and *Constraints/Facilitators* is *low* and *Preference* is *low* then *Suitability of Telecommuting for 0 days* is *high*
2) If *Drives* is *middle* and *Constraints/Facilitators* is *middle* and *Preference* is *middle* then *Suitability of Telecommuting for 0 days* is *middle*
3) If *Drives* is *high* and *Constraints/Facilitators* is *high* and *Preference* is *high* and then *Suitability of Telecommuting for 0 days* is *low*

Rule matrix can also be expressed as follows:

$$\begin{bmatrix} 1 & 1 & 1 & 3 \\ 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 1 \end{bmatrix} \tag{10}$$

Genetic algorithm optimised the MFs to be used for the FN as shows below in Figure A2.8 (Appendix 2). Figure A3.8 (Appendix 3) shows the optimised MFs as output of generic algorithm after around 300 generations.

5-4-1-9-2-Suitability of Telecommuting for 1 Day

To look at suitability of telecommuting for 1 day, the observation values will be 35, 95, 65 and 5. Data for inputs are also gathered from tuned model in previous layer. Genfis3 releases the initial MFs to be used by GA as Figure A1.9 (Appendix 1) shows.

Extracted rules are:

1) If *Drives* is *low* and *Constraints/Facilitators* is *middle* and *Preference* is *low* then *Suitability of Telecommuting for 1 days* is *high*
2) If *Drives* is *middle* and *Constraints/Facilitators* is *low* and *Preference* is *middle* then *Suitability of Telecommuting for 1 days* is *middle*

3) If *Drives* is *high* and *Constraints/Facilitators* is *high* and *Preference* is *high* and then *Suitability of Telecommuting for 1 days* is *low*

Rule matrix can also be expressed as follows:

$$\begin{bmatrix} 1 & 2 & 1 & 3 \\ 2 & 1 & 2 & 2 \\ 3 & 3 & 3 & 1 \end{bmatrix} \quad (11)$$

Using the above extracted rules and initial MFs got from Genfis3, optimised MFs are obtained using GA (Figure A2.9 – Appendix 2 and Figure A3.9 – Appendix 3).

5-4-1-9-3-Suitability of Telecommuting for 2 Days

Suitability of telecommuting for 2 days is examined with observation values 5, 35, 95 and 65 for chosen alternative 0, 1, 2 and 3. Figure A1.10 (Appendix 1) shows the initial Gaussian MFs obtained from Genfis3 function.

Extracted rules are:

1) If *Drives* is *low* and *Constraints/Facilitators* is *low* and *Preference* is *low* then *Suitability of Telecommuting for 2 days* is *low*
2) If *Drives* is *middle* and *Constraints/Facilitators* is *high* and *Preference* is *high* then *Suitability of Telecommuting for 2 days* is *middle*
3) If *Drives* is *high* and *Constraints/Facilitators* is *middle* and *Preference* is *middle* and then *Suitability of Telecommuting for 2 days* is *high*

Rule matrix can also be expressed as follows:

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 3 & 3 & 2 \\ 3 & 2 & 2 & 3 \end{bmatrix} \quad (12)$$

To get optimised MFs, GA is used as Figure A2.10 (Appendix 2) and A3.10 (Appendix 3) show.

5-4-1-9-4-Suitability of Telecommuting for 3 Days

The same way as explained for tuning the final node for alternative 0, 1 and 2, the last node is optimised for alternative 3. Figure A1.11 (Appendix 1) shows the initial inputs and output MFs using Genfis3 function.

Rules for the last alternative and final node are extracted form Genfis3 function.

1) If *Drives* is *low* and *Constraints/Facilitators* is *low* and *Preference* is *low* then *Suitability of Telecommuting for 3 days* is *low*
2) If *Drives* is *middle* and *Constraints/Facilitators* is *middle* and *Preference* is *middle* then *Suitability of Telecommuting for 3 days* is *middle*
3) If *Drives* is *high* and *Constraints/Facilitators* is *high* and *Preference* is *high* and then *Suitability of Telecommuting for 3 days* is *high*

Rule matrix can also be expressed as follows:

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 3 \end{bmatrix} \tag{13}$$

Membership functions are optimised using GA as shown in Figures A2.11 (Appendix 2) and A3.11 (Appendix 3).

5-4-2-Horzental/Vertical Merging in Fuzzy Network

As explained above, all the important external inputs, intermediate variables and branches in the proposed network have been selected (Figure 5.25). Also, all the rules for subsystems were extracted and membership functions were optimised.

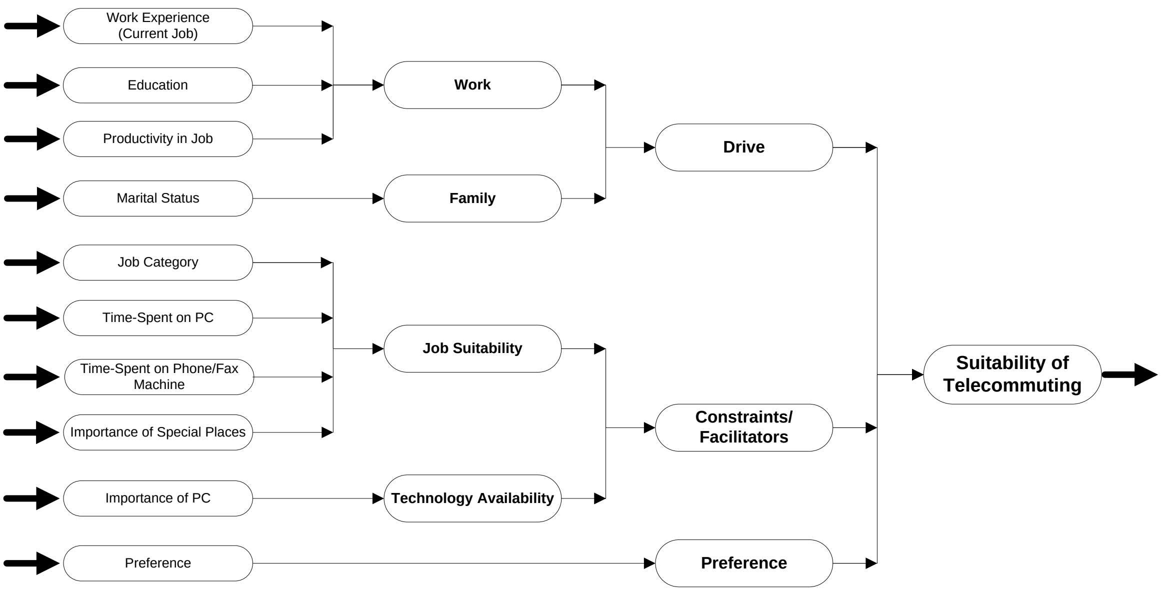

Figure 5.25: Optimised Network

Figure 5.58 shows that only 10 external inputs have significant influence on the final output. It also illustrates that only variables work and family (marital status) are main drives for adopting telecommuting. Suitability of job and availability of technology, which depends on the importance of PC, are either constraint or facilitator to telecommute.

Linguistic composition approach is implemented to simplify the above rule based network. All modular bases and nodes are expressed as Boolean matrices and using horizontal and vertical merging techniques as binary operations and also the proposed efficient method in merging rule bases (see 3-6-2-5), discussed in chapter 3, the networked rule bases are simplified to a linguistic equivalent single rule base.

The single equivalent rule bases for alternatives are obtained as shown below. The external inputs are: *Work experience (current job), Education, Ideology about productivity in job, Marital status, Job category, Time spent on PC, Time spent on Phone/Fax, Importance of special places, Importance of PC and preferences.* In the matrices below, columns 1-10 are the external inputs and column 11 is the output as shown in equation (1). 1,2 and 3 represent low, middle and high respectively.

Suitability of telecommuting for 0 day:

$$\begin{bmatrix} 3 & 2 & 3 & 3 & 1 & 3 & 3 & 2 & 3 & 3 & 1 \\ 2 & 3 & 2 & 2 & 2 & 2 & 1 & 1 & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 & 3 & 1 & 2 & 3 & 1 & 1 & 3 \end{bmatrix} \tag{14}$$

Suitability of telecommuting for 1 day:

$$\begin{bmatrix} 3 & 2 & 3 & 3 & 1 & 3 & 3 & 2 & 3 & 3 & 1 \\ 2 & 3 & 2 & 2 & 3 & 1 & 2 & 3 & 1 & 2 & 2 \\ 1 & 1 & 1 & 1 & 2 & 2 & 1 & 1 & 2 & 1 & 3 \end{bmatrix} \tag{15}$$

Suitability of telecommuting for 2 days:

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 3 & 1 & 2 & 3 & 1 & 1 & 1 \\ 2 & 3 & 2 & 2 & 1 & 3 & 3 & 2 & 3 & 3 & 2 \\ 3 & 2 & 3 & 3 & 2 & 2 & 1 & 1 & 2 & 2 & 3 \end{bmatrix} \tag{16}$$

Suitability of telecommuting for 3 days:

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 3 & 1 & 2 & 3 & 1 & 1 & 1 \\ 2 & 3 & 2 & 2 & 2 & 2 & 1 & 1 & 2 & 2 & 2 \\ 3 & 2 & 3 & 3 & 1 & 3 & 3 & 2 & 3 & 3 & 3 \end{bmatrix} \tag{17}$$

All the requirements for Fuzzy Network modelling are prepared. The utility of alternatives (systemic part of utility function) for each individual is obtained from FN model. Having a single equivalent rule base for each alternative and optimised membership functions, only one single sequence of Fuzzification, Implication and Defuzzification is needed to get the utility values. External inputs for both training and test data sets are fed to the model and different performance indicators are discussed in Section 5-7. Having the utilities of all alternatives for individuals, probably choice can be easily calculated mentioned approach in chapter 3 (Luce, 1959; Quattrone & Vietetta, 2011; Tversky, 1972). The highest probable alternative is considered as employee's choice.

The accuracy of the FN model is examined by comparing the observation against model prediction. The highest probability for alternatives for each individual is considered as chosen alternative and matrices of observation and prediction are generated. These matrices are produced for both trained and tested data. The main diagonal of the matrices where model prediction is the same with observation shows the percentage correct as a measure of model prediction.

Training data:

$$\begin{array}{l} \text{Prediction} \longrightarrow \\ \text{Observation} \\ \quad\big\downarrow \end{array} \qquad \begin{array}{c c} & \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 18 & 5 & 5 & 2 \\ 12 & 17 & 18 & 11 \\ 2 & 2 & 12 & 4 \\ 0 & 3 & 4 & 10 \end{bmatrix} \end{array} \tag{18}$$

Testing Data:

$$\begin{array}{l} \text{Prediction} \longrightarrow \\ \text{Observation} \\ \quad\big\downarrow \end{array} \qquad \begin{array}{c c} & \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 15 & 2 & 7 & 4 \\ 7 & 9 & 20 & 18 \\ 3 & 4 & 4 & 6 \\ 1 & 4 & 6 & 10 \end{bmatrix} \end{array} \tag{19}$$

Percent correct is calculated for both training and testing data for RBN:

Training data: $\frac{18+17+12+10}{125} \times 100 = 46\%$

Testing data: $\frac{15+9+4+10}{120} \times 100 = 32\%$

Also, accuracy of the model for both training and testing data set is achieved by finding the average distance between observation and prediction for all the sample using formula 20 where Y_i is the choice observation of individual *i*, y_i is model prediction of individual *i* and *n* is total number of individual:

$$A = \frac{|Y_i - y_i|}{n} \tag{20}$$

Training data: A= 1.074, Testing data: A= 1.35

5-5-Modelling Suitability of Telecommuting Using Standard Fuzzy System

As explained before in chapter 2, rule based network is a hybrid approach between Standard Fuzzy System and Chained/Hierarchical Fuzzy System. In other words, RBN works as a transparent CSF or HFS and also the same as SFS has only one sequence of Fuzzification, Implication and Defuzzification. In previous section, it was shown how to optimise and simplify the RBN and also equivalent linguistic rule for alternatives were figured out.

Equivalent single rule in RBN, make the modelling process similar to Standard Fuzzy System. In order to compare these two systems, a SFS is developed. SFS is a black box model and all the external inputs are fed to the model with no transparency of internal relationships. In this case study, due to large number of external inputs, the model would be so complex. There are several methods for input and feature selection for SFS which is out of the scope of this research. But in order to look at the performance of RBN, selected inputs for that method is used for SFS.

In the same approach discussed in previous section, the SFS is tuned. First, initial Gaussian membership functions as an initial solution for 10 external inputs (selected for RBN using backward selection methods for subsystems) and rules based on fuzzy c-mean are obtained using Genfis3 function of MATLAB. Then using genetic algorithm and training data set, and having the rules and also one possible initial solution, the MFs are optimised. Matrices of rules for all the alternatives are shown below:

Suitability of telecommuting for 0 day:

$$\left[\begin{array}{cccccccccc|c} 2 & 3 & 2 & 3 & 1 & 3 & 3 & 2 & 3 & 3 & 1 \\ 3 & 2 & 3 & 2 & 2 & 2 & 1 & 1 & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 & 3 & 1 & 2 & 3 & 1 & 1 & 3 \end{array}\right] \tag{21}$$

Suitability of telecommuting for 1 day:

$$\left[\begin{array}{cccccccccc|c} 2 & 3 & 3 & 3 & 1 & 3 & 3 & 2 & 3 & 3 & 1 \\ 1 & 1 & 2 & 1 & 3 & 2 & 1 & 3 & 1 & 1 & 2 \\ 3 & 2 & 1 & 2 & 2 & 1 & 2 & 1 & 2 & 2 & 3 \end{array}\right] \tag{22}$$

Suitability of telecommuting for 2 days:

$$\left[\begin{array}{cccccccccc|c} 1 & 1 & 2 & 1 & 3 & 1 & 2 & 3 & 1 & 1 & 1 \\ 2 & 3 & 1 & 3 & 1 & 3 & 3 & 2 & 3 & 3 & 2 \\ 3 & 2 & 3 & 2 & 2 & 2 & 1 & 1 & 2 & 2 & 3 \end{array}\right] \tag{23}$$

Suitability of telecommuting for 3 days:

$$\left[\begin{array}{cccccccccc|c} 1 & 1 & 1 & 1 & 3 & 1 & 2 & 3 & 1 & 1 & 1 \\ 3 & 2 & 3 & 2 & 2 & 2 & 1 & 2 & 2 & 2 & 2 \\ 2 & 3 & 2 & 3 & 1 & 3 & 3 & 2 & 3 & 3 & 3 \end{array}\right] \tag{24}$$

As can be seen above, the rules are slightly different with the equivalent rules obtained for RBN using linguistic composition approach. It simply shows the influence of intermediate variables in getting the equivalent rules for RBN as they are ignored in SFS. The optimised MFs for SFS are also shown in Appendix.

The tuned models for alternatives are used to find the utility of each alternative for individuals and then choice probability of alternatives per employee is calculated using multinomial logit.

Training data:

$$\begin{array}{c} \text{Prediction} \longrightarrow \\ \text{Observation} \\ \downarrow \end{array} \quad \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 20 & 2 & 3 & 2 \\ 10 & 20 & 22 & 5 \\ 0 & 1 & 3 & 4 \\ 2 & 4 & 11 & 16 \end{bmatrix} \end{array} \tag{25}$$

Testing Data:

$$\begin{array}{c} \text{Prediction} \longrightarrow \\ \text{Observation} \\ \downarrow \end{array} \quad \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 12 & 3 & 5 & 2 \\ 9 & 10 & 17 & 19 \\ 2 & 1 & 2 & 2 \\ 3 & 5 & 13 & 15 \end{bmatrix} \end{array} \tag{26}$$

Percent correct is calculated for both training and testing data for SFS:

Training data: $\frac{20+20+3+16}{125} \times 100 = 47\%$

Testing data: $\frac{12+10+2+15}{120} \times 100 = 32\%$

Also, the same approach for FN, the accuracy of the model using A indicator is obtained:

Training data: A= 1.035, Testing data: A= 1.35

5-6-Modelling Suitability of Telecommuting Using Random Utility Models

Random utility models are the bench mark for discrete choice modelling. In order to examine the performance of RBN model for modelling the suitability of telecommuting, the multinomial logit model is developed using NLOGIT software. As explained in chapter 3, this model is calibrated using Maximum likelihood approach.

RUMs are black box nature models which external inputs are fed into the model and their influence on output is determined. In this approach, the output of the model is a vector of coefficients which is calibrated by maximum likelihood method to estimate the values of coefficients for which the observed sample is most likely to have occurred.

To develop a model for suitability of telecommuting, the correlation analysis is undertaken first. It helps to understand the relationship between inputs and also inputs and outputs. Variables are selected to have high correlation with output and also no correlation with other inputs, it also helps lead to prevent multicollinearity happen in modelling. Feed forward approach is implemented to find the suitable and reasonable set of inputs. Unknown parameters also examined using the t-test to see their whether the difference is statistically significant with zero or not. If the coefficients are not statistically significant, they are removed from the model.

The model is developed with around 40 variables and 8 inputs are selected using feedforward approach and significance of their coefficients. Table 1 shows the variables, coefficients, t-test and P-value. As can be seen in Table 1, there are no values for alternative 0 days telecommuting. Nlogit, considers the coefficients of variables for the first alternative (0 days of doing telecommuting) zero as a bench mark and calculate the other coefficients relative to 0.

Model goodness of fit can be assessed by value of ρ^2 which is analogous to R^2 be used in regression. This value (ρ^2) is 0.21, which shows relatively good fitness. Hensher and Stopher (Hensher & Stopher, 1979) justify the that values 0.2 to 0.4 for ρ^2 represent an excellent fit for behavioural travel modelling. Therefore, calibrated multinomial logit model is a reliable model and can be used as a good bench mark. Also, Domencich and McFadden (1975) have presented a relatively stable empirical relationship between ρ^2 and R^2 in regression, which shows 0.21 roughly equals to 0.6 in scale of R^2 which can be interpreted as a reasonable goodness of fit for a model. In addition, as explained in chapter 3, $-2[L(0) - L(\beta)]$ is 73.76 which is greater than critical χ^2 from the Chi-square distribution at 5 percent level of significance (32.67 for 21 degree of freedom) which shows the whole set of variables are statistically significant.

Table 5.1- Results for Multinomial Logit model for suitability of telecommuting

Variable	Coefficient	T-test	P-Value
Suitability of 1 day telecommuting			
Work experience (current job)	0.230	2.56	0.01
Job Category	-0.228	-1.59	0.11
Preference	0.488	2.03	0.04
Productivity in job	-0.658	-1.80	0.07
Time spent on PC	0.232	1.71	0.08
Time spent on Phone/Fax	0.191	0.91	0.37
Using organisation's car	-0.414	-0.83	0.40
Time spent with Colleagues/Clients	-0.201	-1.03	0.20
Suitability of 2 days telecommuting			
Work experience (current job)	0.288	3.18	0.00
Job Category	-0.273	-1.96	0.05
Preference	0.865	3.61	0.00
Productivity in job	0.079	0.27	0.82
Time spent on PC	0.100	0.77	0.44
Time spent on Phone/Fax	-0.068	-0.30	0.76
Using organisation's car	-0.930	-1.82	0.06
Time spent with Colleagues/Clients	-0.299	-1.44	0.14
Suitability of 3 days telecommuting			
Work experience (current job)	0.248	2.40	0.01
Job Category	-0.401	-2.42	0.01
Preference	1.281	4.54	0.00
Productivity in job	-0.550	-1.39	0.16
Time spent on PC	0.384	2.50	0.01
Time spent on Phone/Fax	0.200	0.82	0.41
Using organisation's car	-1.644	-2.62	0.00
Time spent with Colleagues/Clients	-0.446	-1.86	0.06
Number of observation	125		
Degree of freedom	21		
$L(\beta)$	-136.41		
$L(0)$	-173.29		
ρ^2	0.21		

Calibrated model shows that *Work experience (in current job), Job Category, Preference, Productivity in job, Time spent on PC, Time spent on Phone/Fax, Using organisation's car* and *Time spent with Colleagues/Clients* are the important factors that influence on the output and selecting alternatives.

Work experience (in current job) has positive impact and having more experience leads to selecting 2 days of telecommuting. As expected, *Preference* has positive coefficients and increases from alternative 0 to 3 which shows the direct relationship between preference and days of adopting telecommuting. *Job category* has negative coefficient and shows in job groups (chapter 4) in that category are more suitable for more days of doing telecommuting. *Using organisation's car* as expected has negative influence on output and using organisation's car less encourage the employees to do telecommuting.

If employees spent more time with colleagues/clients, telecommuting is less suitable as coefficient of *Time spent with Colleagues/Clients* shows. For *Productivity in job* having strong ideology makes 2 days of telecommuting more suitable. Also, for *Time spent on PC* and *Time spent on Phone/Fax* there is no clear pattern but spending more time on these two factor lead to do more days of telecommuting.

The coefficients are used for finding the systematic part of utility functions and eventually probability choice for individual. Matrices of observation and prediction for both training and testing data are shown below:

Training data:

$$\text{Prediction} \longrightarrow \quad \text{Observation} \downarrow \quad \begin{array}{c c} & \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 20 & 3 & 9 & 0 \\ 8 & 5 & 8 & 6 \\ 2 & 4 & 24 & 9 \\ 2 & 2 & 10 & 13 \end{bmatrix} \end{array} \tag{27}$$

Testing Data:

$$\text{Prediction} \longrightarrow \quad \text{Observation} \downarrow \quad \begin{array}{c c} & \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 12 & 5 & 5 & 4 \\ 8 & 3 & 4 & 4 \\ 8 & 8 & 13 & 8 \\ 5 & 3 & 19 & 11 \end{bmatrix} \end{array} \tag{28}$$

Percent correct is calculated for both training and testing data for RUM:

Training data: $\frac{20+5+24+13}{125} \times 100 = 49\%$

Testing data: $\frac{12+3+13+11}{120} \times 100 = 32\%$

Also, the same approach for FN and SFS, the accuracy of the model using A indicator is obtained for RUM:

Training data: A= 1.04, Testing data: A= 1.34

5-7-Models Comparison

Suitability of telecommuting has been modelled through 3 models. In RBN, a transparent framework was modelled where all the external inputs, intermediate variables and connections between layers and levels were discussed. Rules for subsystems were obtained and whole mode can be interpreted clearly. Considering all the modular bases and subsystems, equivalent rules for alternatives were identified. For each alternative, 3 rules were extracted from the training data set. 10 external inputs were also selected, important intermediate variable were kept into the model and non-significant branches of the network were removed. Those inputs were used for SFS, and 4 sets of rules extracted for 4 alternatives. As a bench mark model, multinomial logit model using random utility approach was also developed. 8 inputs were selected for RUM model.

Between selected and important variables for FN, SFS and RUM, 6 variables are common which are *Work experience (current job), Job Category, Preference, Productivity in job, Time spent on PC, Time spent on Phone/Fax*. That shows the importance of these 6 feature is suitability of telecommuting. That shows relatively more than 60 percent similarity in finding inputs that have significant influence on the output.

To compare these three models, various performance indicators are used and explain in the following sections.

5-7-1-Accuracy

Accuracy of each model has already been discussed in relevant sections. Table 2 has integrated all those values to have better understanding to this performance indicator.

Percent correct:

Table 5.2 - Accuracy indicator: Percent correct (percentage)

	RBN	SFS	MNL
Training data	46	47	49
Testing data	32	32	32

Average distance:

Table 5.3- Accuracy indicator: Average distance

	RBN	SFS	MNL
Training data	1.07	1.04	1.04
Testing data	1.35	1.35	1.34

From above two indicators for training and testing data, accuracy of models is almost the same (very small differences can be neglected).

In addition, Figures 5.26-5.65 compare models’ prediction and observation for both training and test data sets for MNL, SFS and FN.

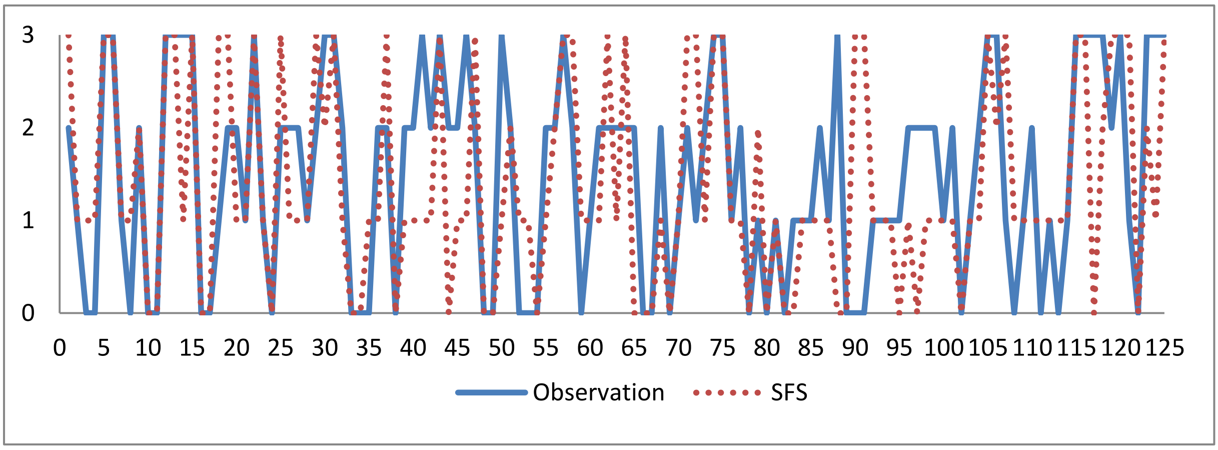

Figure 5.26: SFS model prediction and observation for training data (individual vs choice)

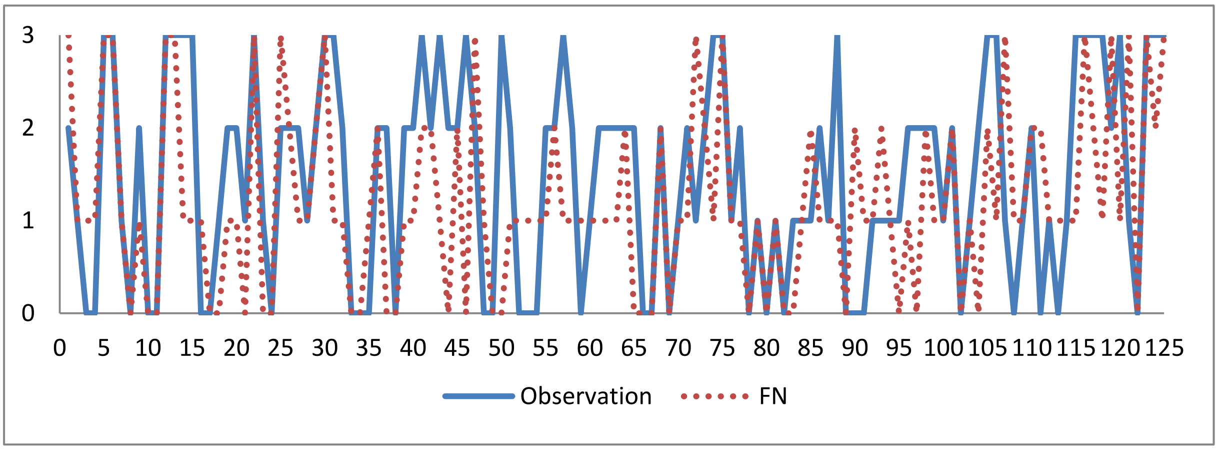

Figure 5.27: FN model prediction and observation for training data (individual vs choice)

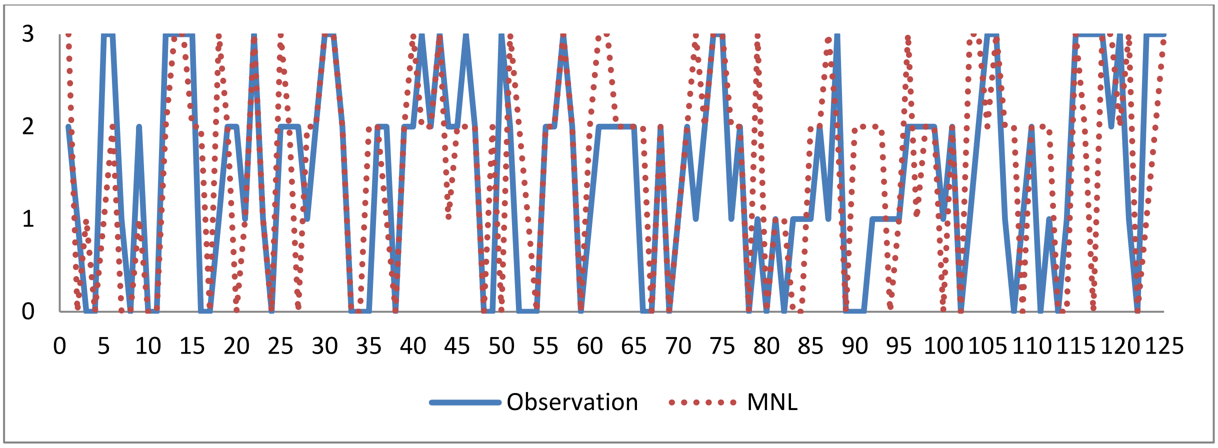

Figure 5.28: MNL model prediction and observation for training data (individual vs choice)

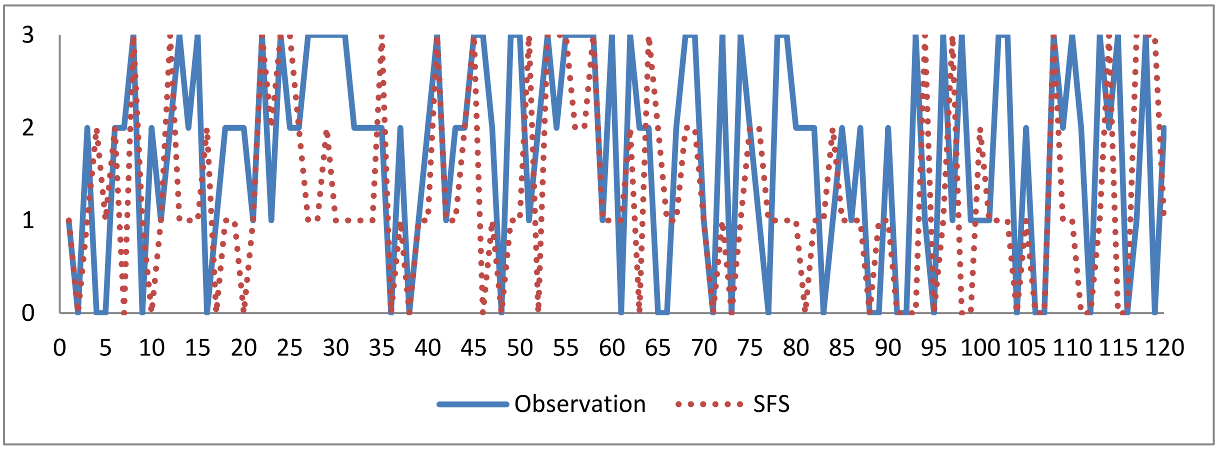

Figure 5.29: SFS model prediction and observation for test data (individual vs choice)

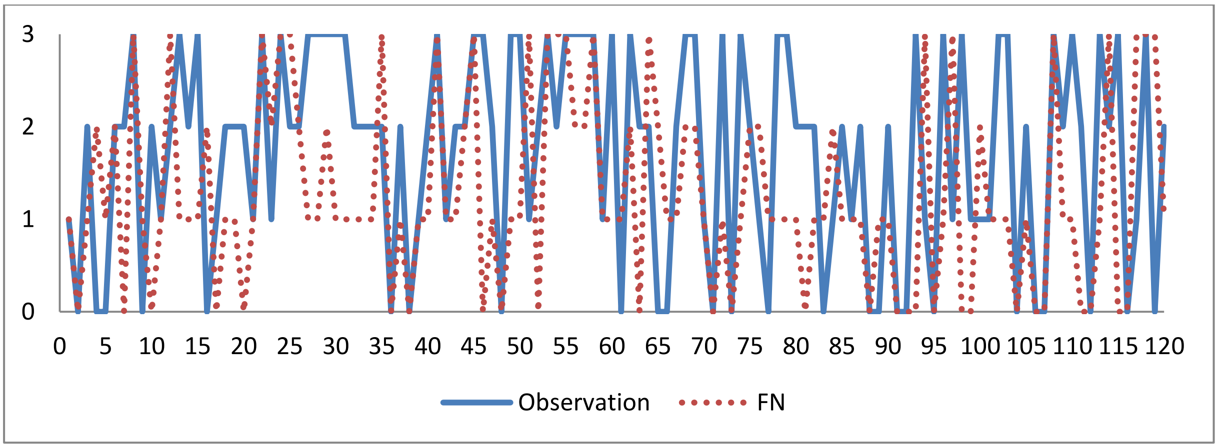

Figure 5.30: FN model prediction and observation for test data (individual vs choice)

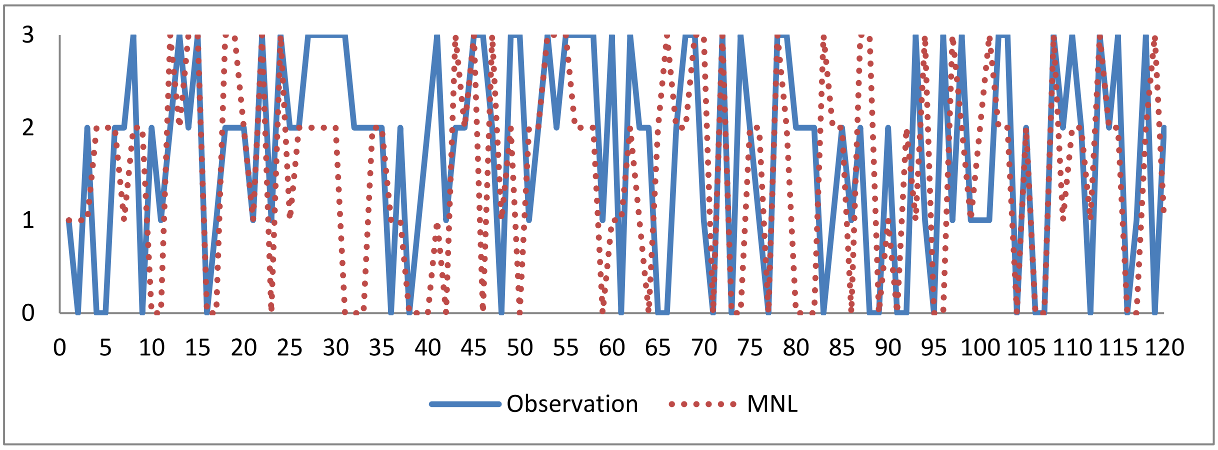

Figure 5.31: MNL model prediction and observation for test data (individual vs choice)

5-7-2-Transparency

As explained in methodology chapter, transparency index defined as:

$$T = \frac{p+q}{n+m} \tag{29}$$

Where:

n: Number of nodes
m: Number of connections
P: number of inputs
q: number of outputs

For the above models, table 4 compare transparencies. Lower value for this index shows the higher transparency.

Table 5.4- Transparency indicator

RBN	SFS	MNL
$\frac{10+4}{8+8} = 0.87$	$\frac{10+4}{1+0} = 14$	$\frac{8+4}{1+0} = 12$

From the above table, RBN network as expected has the highest transparency and SFS lowest due to having more inputs in comparison with MNL.

5-7-3-Efficiency

Since explained in methodology chapter, efficiency is examined from perspective of number of times that models are developed, tuned and calibrated.

For Rule based Network Model:

$$E_{RBN} = \sum_{j=1}^{m} \sum_{i=1}^{n} (\frac{x_{ij}(x_{ij}+1)}{2} - 1) + mn \tag{30}$$

Where

m: Number of layers
n: Number of levels
x_{ij}: Number of inputs in layer i and level j in non-identity nodes

For Multinomial Logit Model:

$$E_{MNL} = \frac{x(x+1)}{2} \tag{31}$$

where x is the number of inputs.

Table 6, compare the efficiency of 3 models using above mentioned indicator.

Table 5.5- Efficiency indicator– Number of modeling practices

RBN	SFS	MNL
164	4	496

For modelling SFS, the selected inputs for RBN were used and model was developed. Therefore, it cannot be an accurate indicator for that model. Table 6 shows that RBN is more efficient than MNL in terms of number of times for developing, tuning and calibrating different parameters of each model.

5-7-4-Interpretability

In methodology chapter interpretability indicator was explained. This indicator roughly explains how the models can be interpreted using models specifications. Simple formulas have been introduced in chapter 3:

$$I_{RBN\ \&\ SFS} = N_a . \sum_n N_r . N_l . N_x \tag{32}$$

where N_r is the number of the rules, N_l is number of linguistic terms , N_x is number of inputs and number of outputs (alternatives) N_a and n is number of non-identity nodes where equivalent linguistic rules also considers as a node.

$$I_{MNL} = N_x . N_a \tag{33}$$

N_x is number of variables and their coefficients and N_a is the number of alternatives.

Table 5.6- Interpretability indicator

RBN	SFS	MNL
586	360	32

Table 7 clearly shows the power of linguistic models. For MNL, only coefficients are the means for interpreting the model and it is not easy to see the influence of all variables together and then on the output. But in RBN due to its white-box nature, intermediate variables make the model more transparent and interpretable using linguistic variables. Influence of inputs on the outputs in subsystems is explicitly shown by rules. Finally in equivalent linguistic rules, the way external inputs influence each other and on the final output are expressed by set of linguistic rules which are much more interpretable and close to human decision making process. In SFS, there are 4 alternatives, 10 inputs and 3 linguistic terms which makes in more interpretable than MNL but less than RBN.

5-8-Conclusion

In this chapter suitability of telecommuting was modelled using Fuzzy Networks. Firstly, the existing conceptual framework and internal decision making process for adopting telecommuting was transformed to form of a network to illustrate external input, intermediate variables and their relationship. Subsystems were modelled by extracting data driven rules using clustering approach. Optimised membership functions were produced using Genetic Algorithm. Also, the most important inputs and network branches (statistically significant) were selected using proposed backward selection method.

Obtaining the optimised size of the network, rules for all subsystems and also tuned membership functions led to find the equivalent linguistic rules. Linguistic composition approaches such as horizontal and vertical merging and also the efficient method of merging modular bases helped to achieve the equivalent linguistic rule for each alternative. In final steps, maximum utility of alternatives were achieved using the equivalent linguistic rule and optimised MFs.

As a result of fuzzy network approach (white-box model), important external inputs and intermediate variables as well as linguistic rules for all four alternatives were obtained which help to understand how employees make decision to adopt telecommuting. Also, a standard fuzzy system as the basic black-box fuzzy model was developed to compare the performance of the fuzzy models.

Moreover, a multinomial logit model as benchmark approach in choice modelling were developed to compare the performance of the fuzzy network approach with the established method. Results show that in terms of accuracy, models perform almost the same. However, fuzzy network is more transparent and interpretable.

CHAPTER 6
CONCLUSIONS

6-1-Introduction

Telecommuting as an approach in Transportation Demand Management (TDM) can play an important role in lessening or redistributing peak hour trips. In this study, the suitability of telecommuting has been modelled using Rule Based Network (RBN) and compared with existing approaches via different performance indicators.

6-2-Methodolody

Different conceptual frameworks have been proposed by several scholars for adopting telecommuting and mostly refer telecommuting as a multidimensional problem that includes different aspects of life such as family, job, commuting and so on. Those comprehensive structures not only consider various external inputs, but also highlight the role of intermediate variables, their interactions and also their influence on final output.

Adopting telecommuting is also considered as a choice problem and employees can work from home for number of days per week which suit them best. Therefore, in modelling suitability of telecommuting, a choice model should be developed and calibrated. In other words, to describe how employees make choices between different alternatives, employee's attributes and characteristics should be considered to fit best to an alternative which represent the suitable number of days of working from home.

Choice theory has been widely discussed in the literature and solid theory of random utility is mostly used for choice modelling. In Random Utility Models (RUM), an alternative which has the highest utility is consider as the most probable choice to be selected. In the common approach, utilities are obtained using Maximum Likelihood as a probabilistic approach. Also, That type of model are counted as a black-box model which inputs are fed into the model and output is obtained without considering the interaction of any intermediate variables. Depends on the type of uncertainty and error terms, different type of models can be developed. In case of having few alternatives, Multinomial Logit (MNL) model is well used.

As mentioned above, telecommuting is considered as a multidimensional decision making problem and fuzzy logic is a powerful tool for resembling human decision making process with an ability to generate precise solutions from certain or approximate information. It is believed that employees use linguistic variables to maximise their utilities for different alternatives. Thus, fuzzy logic is considered as a promising approach for obtaining utility of alternatives in order to find choice probabilities.

To consider the impact of intermediate variables and also their interaction, Fuzzy Rule Based Networked has been selected. RBN is characterised by a white-box nature whereby the inputs are mapped to the outputs by means of connections. Subsystems in RBN are represented by nodes and the interactions among subsystems are the connections among these nodes. RBN is a hybrid between Standard Fuzzy System (SFS) and Chained/Hierarchical Fuzzy System (CFS/HFS). On one hand, the structure of RBN is similar to the structure of CFS/HFS due to the explicit presentation of subsystems and the interactions among them. On the other hand, the operation of fuzzy RBN resembles the operation of SFS as the multiple rule bases are simplified to a linguistically equivalent single rule base. This simplification is based on the linguistic composition approach. Transparency of RBN in identifying the crucial role of subsystems and its influence on equivalent linguistic rule are significant advantages of utilising this approach.

To be able to use RBN in this study, the comprehensive internal decision making process for adopting telecommuting is converted to form a network by identifying external inputs, subsystems, connections and output.

To find the maximum utility of each alternative, the model should be calibrated by observations' value. In random utility models such as MNL, model is calibrated and the coefficients of variables are determined to make the model's prediction close to observation. In RBN, as there is no coefficient, Membership Functions (MFs) play similar role to parameters in RUMs to adjust the variables and in order to maximise the utility function, MFs should be optimised. Also, to get optimised utility function, decision rules should be driven from data. Since RBN work on basis of Mamdani system, different optimisation methods such as adaptive network-based fuzzy inference system (ANFIS) for tuning Sugeno-type fuzzy system is not applicable for finding rules and MFs.

In this study, to maximise the utility of alternatives, rules for all the subsystems and also MFs for both external inputs and all the connections should be obtained and optimised. In order to optimise the subsystems, the values of intermediate variables are needed. Every subsystem and its inputs carries the meaning and concept of that intermediate variable. As in the available data set, there is no value for intermediate variables, the observation values were used to calibrate the modular subsystems. That helps to merge the relevant inputs in line with the final output and also see the influence of those variables on the next layer of intermediate variables as well as final output.

To tune the subsystems, Genfis3 function of MATLAB software was used for driving the rules from the data using fuzzy c-mean. Genfis3 also generates Gaussian MFs which are not optimised and it is not possible to use ANFIS method due to Mamdani nature of RBN. In order to optimise the MFs, Genetic Algorithm (GA) was used. In this study, 3 linguistic variables and rules was considered for sake of simplicity, reducing the size of problem and computational comfort. To tune the MFs using GA, Gaussian MFs were approximated with simple triangular MFs and embedded in initial population which helped convergence to happen faster and more efficient.

Dimensionality is another problem with RBN. In this study this issue was tackled by input and branch selection using backward method. This method is based on generating only one tuned fuzzy model, which explained above using clustering and GA, that takes all the possible input variables and then systematically removing antecedent clauses in the fuzzy rules of this initial tuned model and evaluate the performance of the model. Subsystems were tuned using the training data set (125 individuals) and inputs and branches were selected via testing data set (120 individuals). The best model in terms of error criterion determines the number of inputs for the each subsystem. In other words, significant external inputs were selected from the first layer of model and for the next layers, important branches were kept in the model to reduce the size of whole model.

Obtaining the optimised size of the network, rules for all subsystems and also tuned membership functions led to find the equivalent linguistic rule. Linguistic composition approaches such as horizontal and vertical merging and also the efficient method of merging modular bases helped to achieve the equivalent linguistic rule for each alternative. In final steps, maximum utility of alternatives were achieved using the equivalent linguistic rule and optimised MFs.

Fuzzy rule based network, as a novel approach, was used for modelling telecommuting due to its powerful abilities. In order to add more values to this study, a Standard Fuzzy System and also a Multinomial Logit model as Random Utility Model, common approach, were developed for comparison purposes. Worth noting that selected inputs for RBN was used for SFS as primary model in fuzzy system.

6-3-Findings and Results

Models for suitability of telecommuting were developed using novel approaches of fuzzy Rule Based Network as well as SFS and MNL. Models were made using the training data set and evaluated by both training and test data. Most important variables were identified in describing suitability of telecommuting Different performance indicators have been used to compare these approaches.

Accuracy: In terms of accuracy of models for prediction, average distance and percent correct have been used. Percent correct shows the percentage of models' correct prediction and average distance demonstrates the average difference between models' predictions and observations. Accuracy indicators for these three models are almost the same value which clarifies the tuned fuzzy rule based works as good as MNL as benchmark model in choice modelling.

Transparency: One of the most important reasons that RBN has been implemented is this study is its white-box nature. It is a transparent model and not only the external inputs are considered, but also intermediate variables, their connections and interactions between them give a better view to internal decision making process for modelling telecommuting. Transparency index considers the connections plus inputs. Expectedly, RBN is the most transparent approach for modelling telecommuting. SFS and MNL are black-box model, less transparent, and their transparency indices are similar though number of inputs for SFS is more.

Interpretability: In recent years the interest of researchers in obtaining more interpretable models has grown. The challenge has been the trade-off between accuracy and interpretability and in case of having similar accuracy level, more interpretable one is preferable. Fuzzy models are interpretable better by human as they are based on rules and linguistic terms.

RBN, as a white-box model, was used for modelling telecommuting which helped to make the internal decision making process transparent. All the intermediate variables can be interpreted by their specific rules and inputs. It means that not only single equivalent rules for alternatives are interpretable for the whole model and inputs, but also each intermediate variable has its own rules which helps to defining the relationship between variables in middle layers. Also as explained above, SFS as a fuzzy system has good ability for interpreting the relationships between inputs and outputs but still less than RBN due to its black box nature. MNL is interpreted using the coefficients of variables in utility functions which means lower capacity of interpretability. In the other words, as expected, fuzzy based models are more interpretable than other types for modelling due to their linguistic nature.

In this study, for MNL, 8 variables were statistically significant in describing the alternatives for suitability of telecommuting: Work experience (in current job), Job Category, Preference, Productivity in job, Time spent on PC, Time spent on Phone/Fax, Using organisation's car and Time spent with Colleagues/Clients. For different alternatives and due to sign and value of coefficients, different explanations can be made. But in RBN and SFS, Work experience (current job), Education, Ideology about productivity in job, Marital status, Job category, Time spent on PC, Time spent on Phone/Fax, Importance of special places, Importance of PC and Preferences were recognised as important. The relationship between these external variables and also on the final outputs as alternatives are defined by rules and linguistic variables. As mentioned, 6 variables are in common between these models and their behaviours in terms of describing the outputs are similar.

Efficiency: Efficiency is an indicator for the number of times that models are developed, calibrated and tuned. For MNL, feedforward method is commonly used and in case study like telecommuting with large number of external inputs so many iterations are needed. In RBN, modelling and calibrations are repeated in all the subsystems and finally for whole model as a single equivalent rule which includes, using genetic algorithm, input selection and so on. In term of number of time for modelling, RBN is less but regarding computational iterations is not as easy as MNL. Also, due to the fact that selected inputs for RBN was used for SFS, efficiency of that model should be considered with pinch of salt.

At the end, RBN, as a novel approach in fuzzy modelling, shows its abilities in terms of interpretability, transparency and also accuracy for modelling complex systems like modelling suitability of telecommuting. It has got its advantages as a transparent approach for modelling but disadvantages in computational burden complexity should not be neglected.

6-4-Research Contribution

The main contributions of this research can explained in categories below:

- *Modelling Suitability of telecommuting using Fuzzy Rule Based Network*

  Telecommuting as a multidimensional decision making problem was modelled using Fuzzy Rule Based Network. Fuzzy logic as a powerful tool for resembling human decision making process was considered. Also, Rule based Network was utilised for its abilities as white-box model to show not only the influence of inputs on final outputs but also to demonstrate the role of intermediate variables and their interactions. The internal decision making process was converted to a network then used for modelling by Fuzzy Rule Based Network.

- *Choice modelling using Fuzzy Rule Based Network*

  Adopting telecommuting is a choice problem and employees are faced to a choice set which is the number of days per week. In choice theory and utility approach, an alternative with highest utility is the most probable choice to be selected. Rule Based Network was used to find the maximum utility for alternative and finally probability of choice for alternatives.

- *Tuning Rule Based Network using Genetic Algorithm*

  To find the maximum utility for alternatives, Fuzzy Rule Based Network were tuned. To maximise the likelihood of observation happening, all subsystems should be optimised and consequently whole model was tuned. Since there is no unknown parameters like coefficient similar to other regression models, membership functions and rules should be optimised. Also, as RBN works bases on Mamdani fuzzy inference systems, adaptive network-based fuzzy inference system (ANFIS) cannot be used. Therefore, it was proposed to use fuzzy c-mean clustering approach for deriving the rules out of the data (not expert view). Then using the obtained rules, Genetic Algorithm as heuristic approach in optimisation, form the membership functions.

- *Input and branch selection for Rule Based Network*

  One of the main problems with RBN is the dimensionality and the size of the network and a solution was proposed in this research to tackle that issue. In order to reduce the size of the network, important external inputs and intermediate variables (network branch) should be selected. In Rule Based Network, from left to right, external inputs feed the intermediate variables in the next layer and this pattern continues to the output. Backward selection method was proposed for finding important both external and intermediate variables.

  This approach is based on generating only one tuned fuzzy model for every subsystem, (explained above using clustering and GA), that takes all the possible input variables and then systematically removing antecedent clauses in the fuzzy rules of this initial model and evaluate the performance of the model. The best model in terms of error criterion determines the number of inputs for the each subsystem. In other words, significant external inputs are selected from the first layer of model and for the next layers, important branches are kept in the model to reduce the size of the whole model.

- *Efficient method for Horizontal and Vertical merging of Rule Based Network*

  In large networks, horizontal and vertical merging using Boolean matrices and their manipulation are computationally expensive due to sizes of produced matrices. Manipulating Boolean matrices ends up with matrices with large amount of zeros elements that makes the merging process longer and slowly. Very simple method of tagging was introduced to reduce the sizes of matrices by ignoring zeros elements that made horizontal and vertical merging more efficient and faster.

6-5-Future work

Rule Based Network is a novel approach in fuzzy systems which has significant capacity in modelling. RBN can be used in a wide range of application areas where the knowledge or data about the modelled process can be provided in a modular fashion, i.e. for each interacting sub-process by means of individual rule bases. Such modular processes are quite common in many areas such as decision making, manufacturing, communications and transport. In this case, the interacting modules can be decision units, manufacturing cells, communication nodes or traffic junctions. To achieve better results, RBN can be further extended for learning and optimisation of the structure and parameters of fuzzy networks in the context of real-world applications.

Also, the approach can be easily extended to other types of rule based systems such as the ones using deterministic and probabilistic logic. These non-fuzzy rule based systems can be represented by deterministic and probabilistic graphical models, respectively.

Neural Networks can be applied for optimising RBN and its performance can be compared by proposed method in this research. Also dimensionality of RBN can be studied to explore more efficient approaches for reducing the size of the network and speed up modelling process. In other words, efficiency of RBN can be considered as a challenging research area which can be addressed in future researches.

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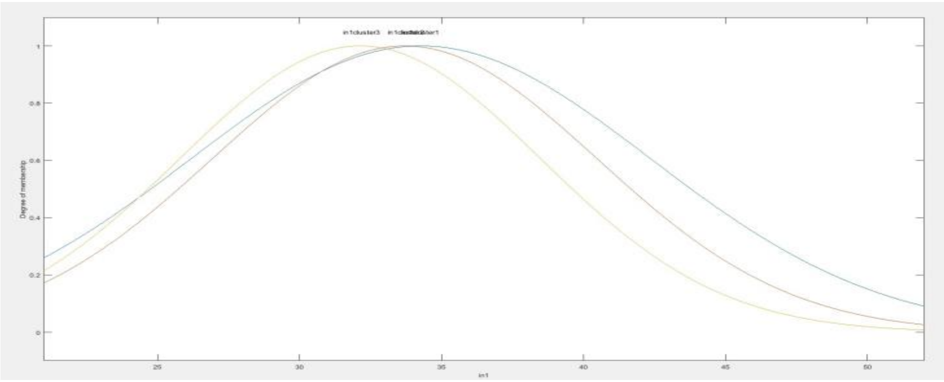
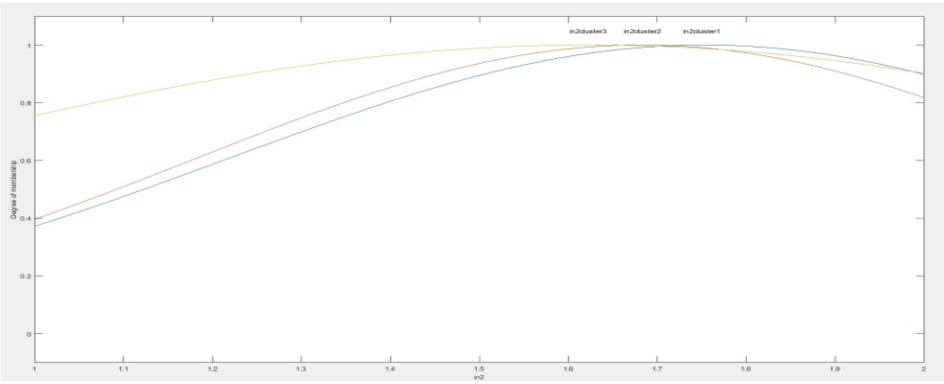
Appendix 1

Initial membership functions are obtained from MATLAB Genfis3 function. The triangular approximation of the obtained membership functions are fed into Genetic Algorithm for tuning and optimisation.

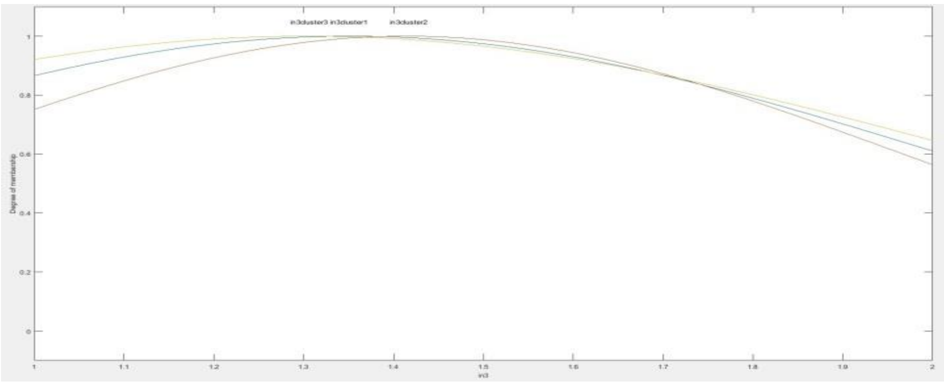
a) Family related drive variables



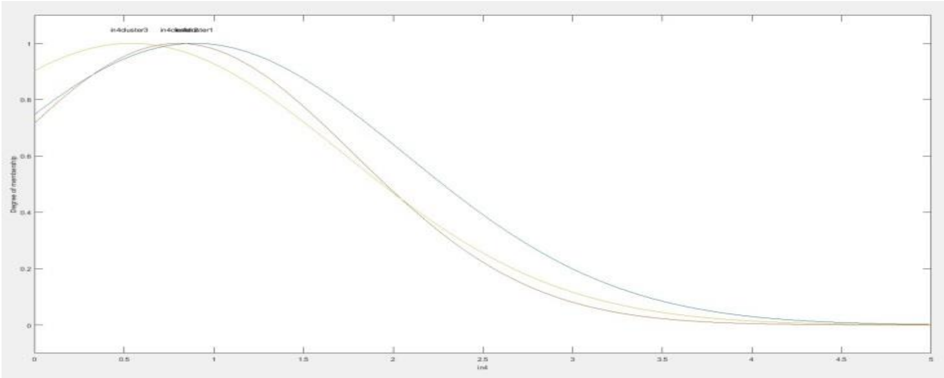
a)



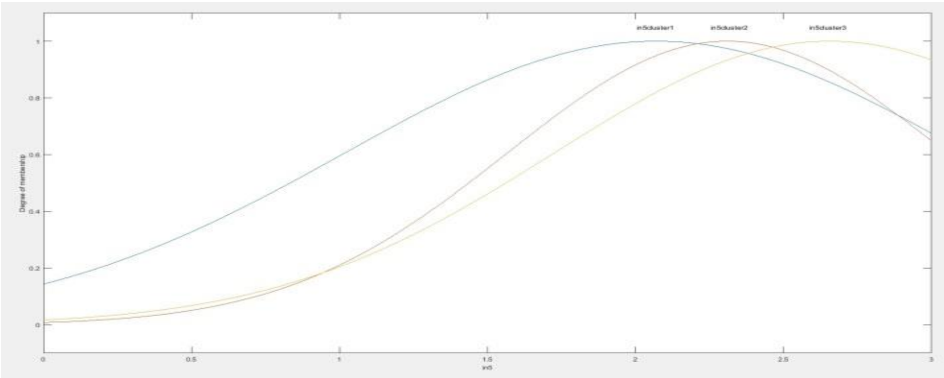
b)



c)



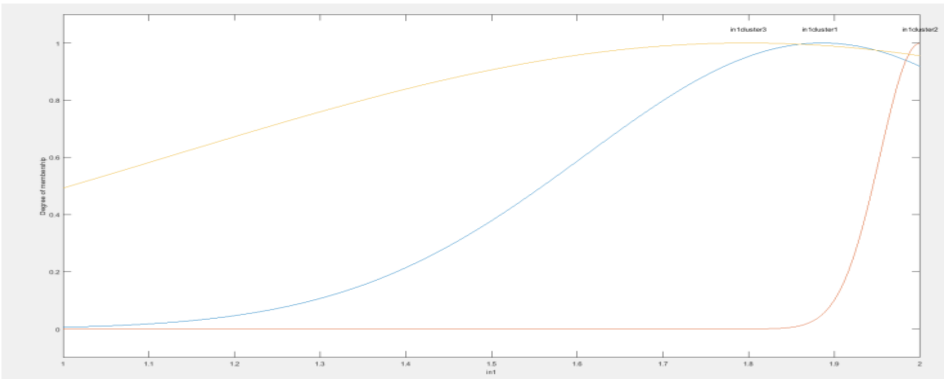
d)



e)

Figure A1.1: Membership functions given by Genfis3 for family related drive variables:
a) Age b) Gender c) Marital Status d) Number of children e) Family wellbeing

b) Commuting related drive variables



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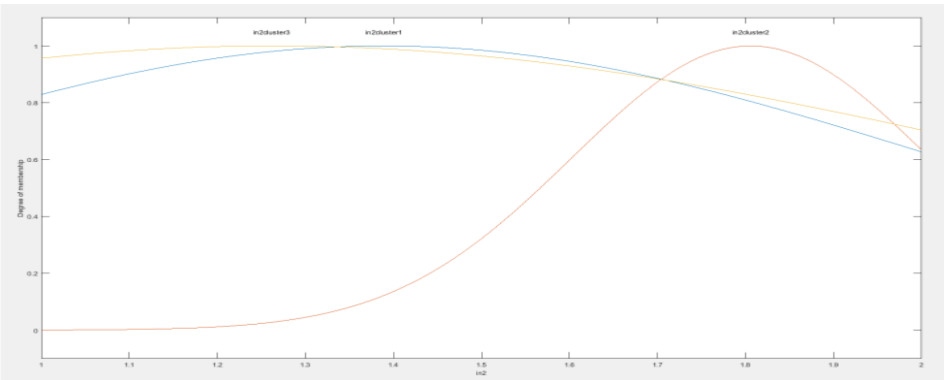

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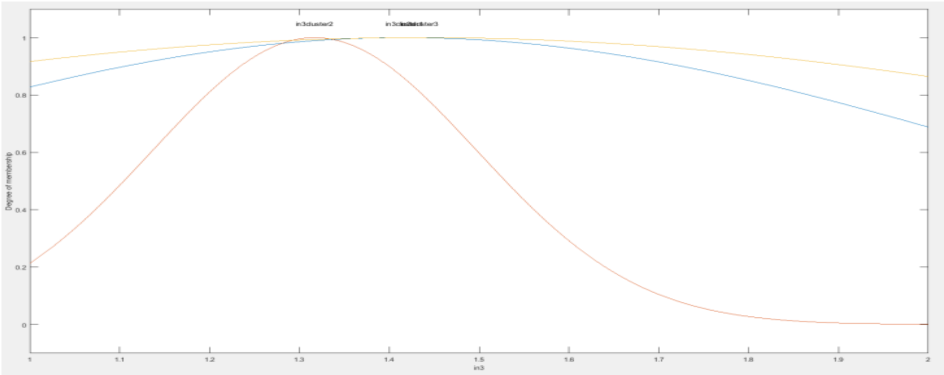
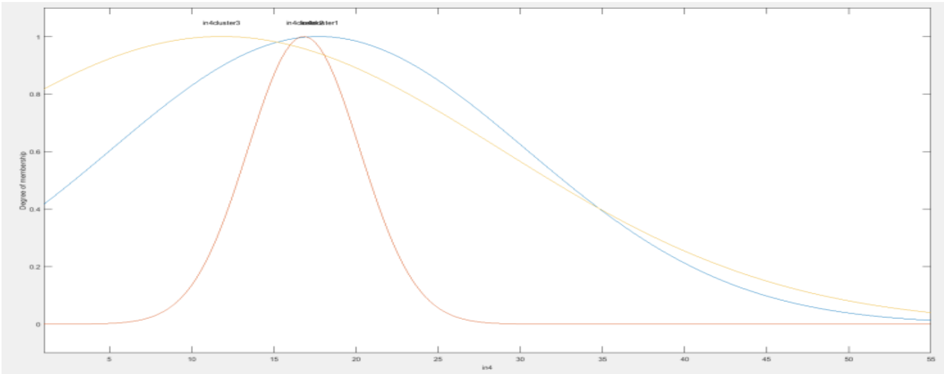
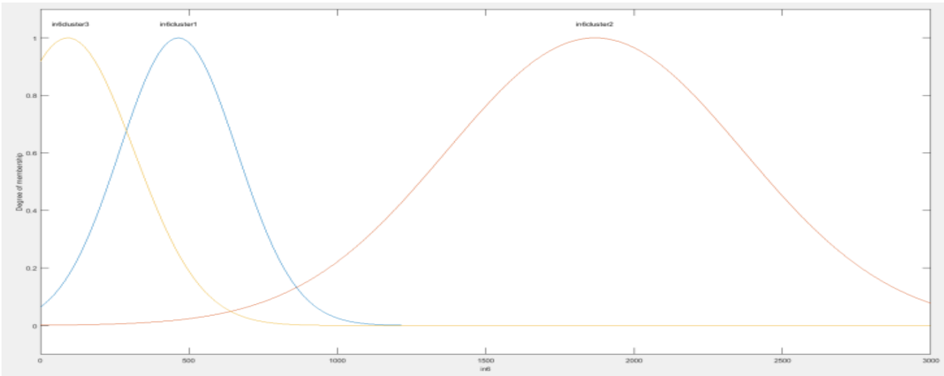

in3cluster2
Degree of membership
in3

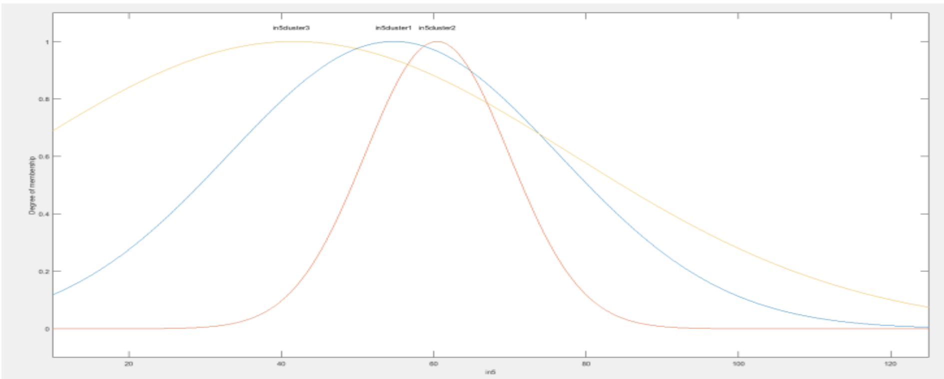
c)


in4cluster3
Degree of membership
in4

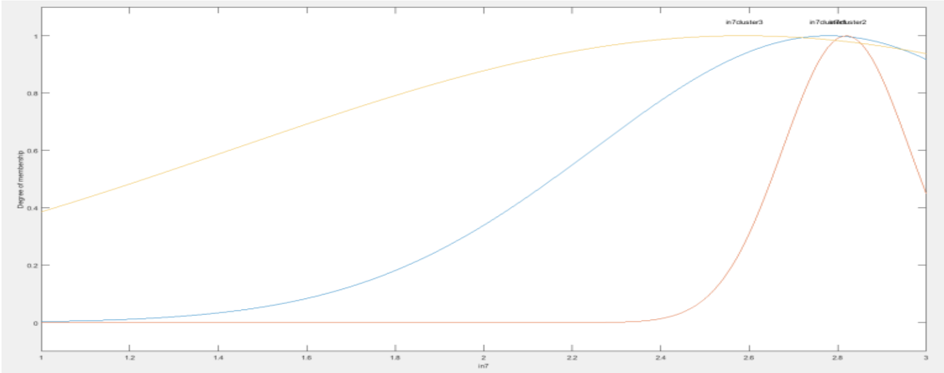
d)


in6cluster3
in6cluster1
in6cluster2
Degree of membership
in6

e)


in5cluster3
in5cluster1
in5cluster2
Degree of membership
in5

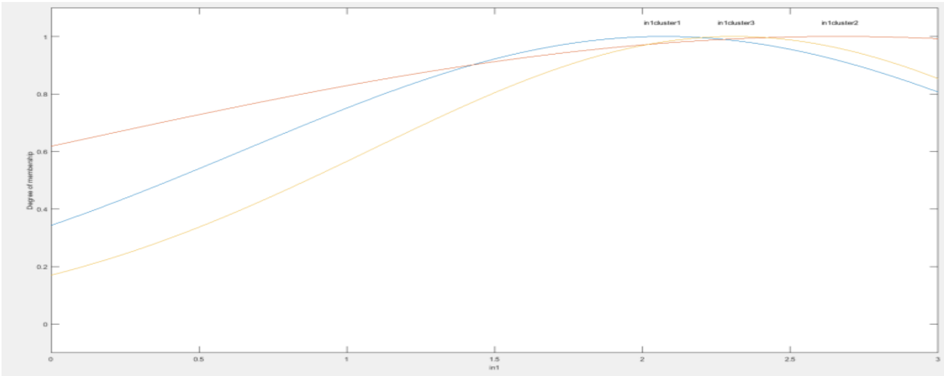
f)



g)

Figure A1.2: Membership functions given by Genfis3 for Commuting related drives variables:
a) Holding driving license b) Car ownership c) Using organisation's car d) Commuting distance e)Travel cost f)Travel time g)Ideology on influence of telecommuting on congestion reduction

c) Ideology related drive variables



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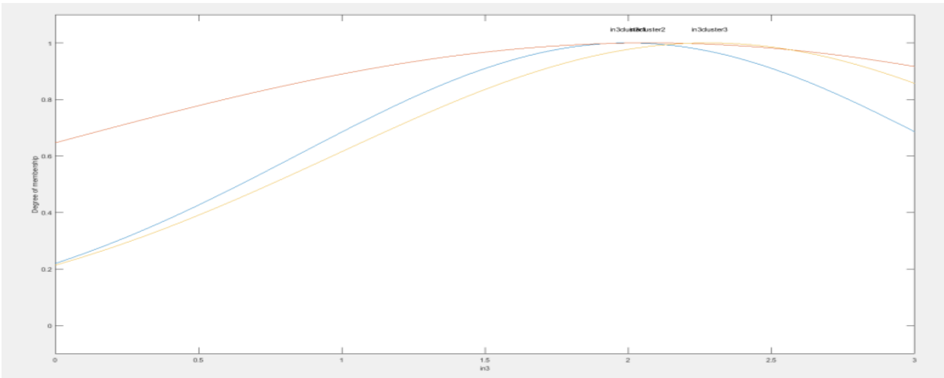

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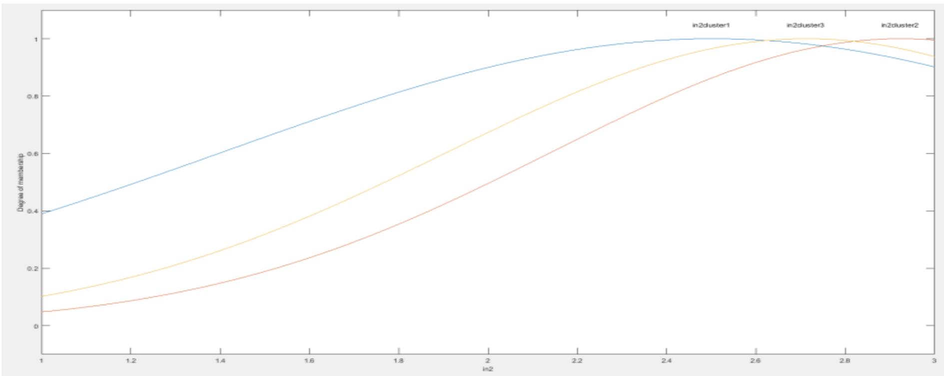
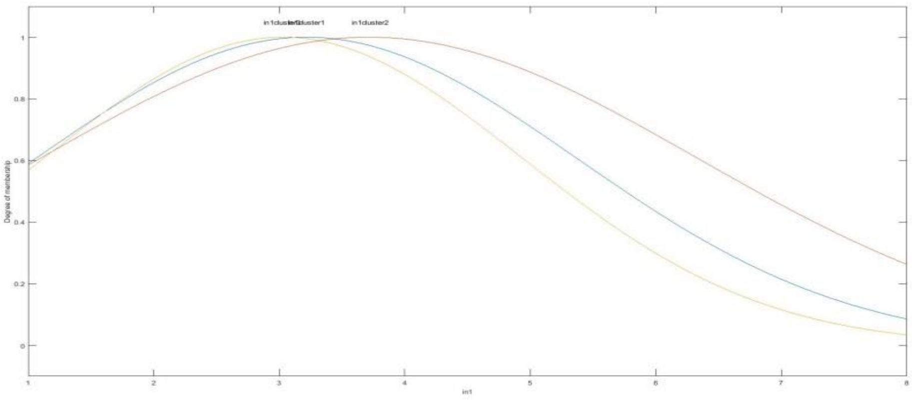


c)

Figure A1.3: Membership functions given by Genfis3 for Ideology related drives variables:
a) Family wellbeing b) Productivity in job c) Congestion reduction

d) Job suitability related Constraints/Facilitators variables



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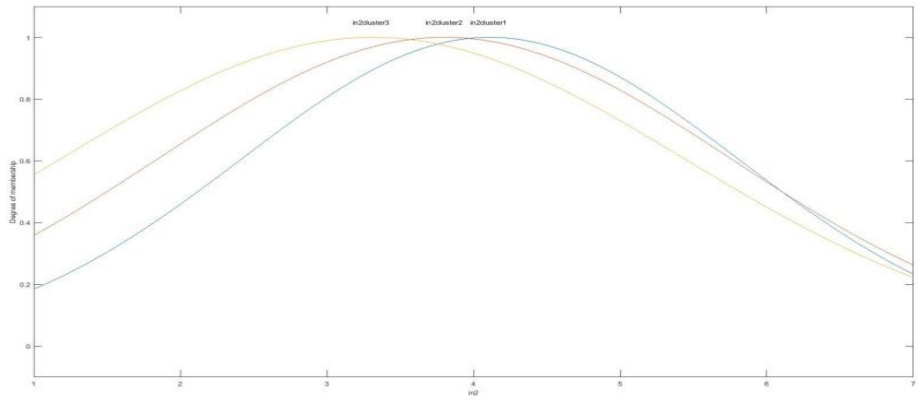

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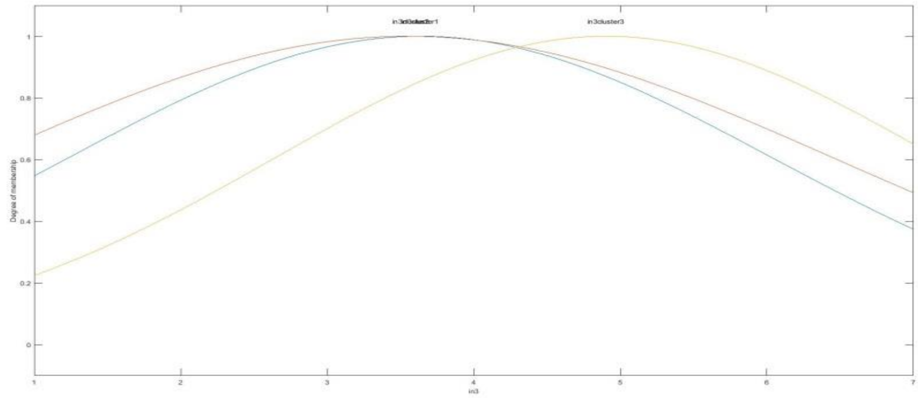

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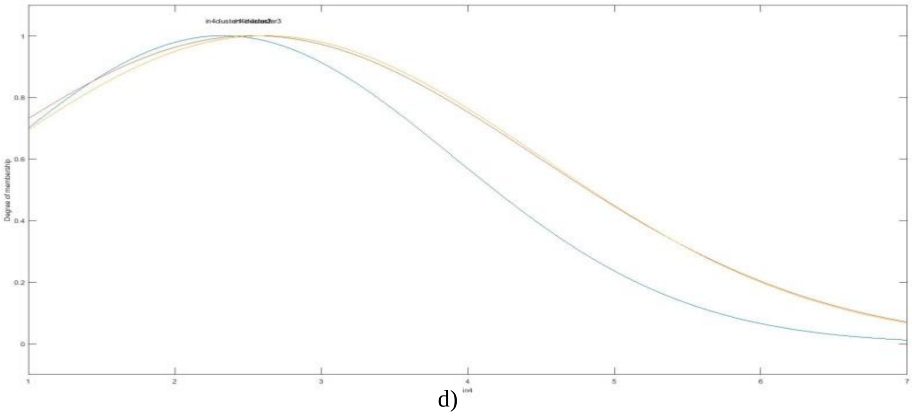

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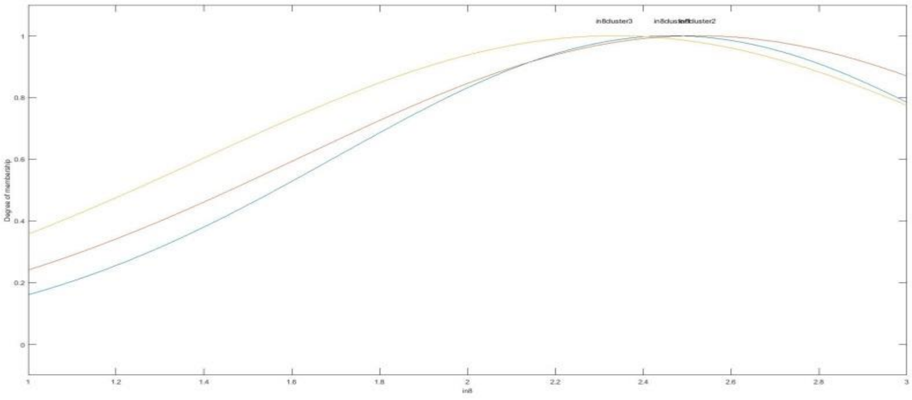
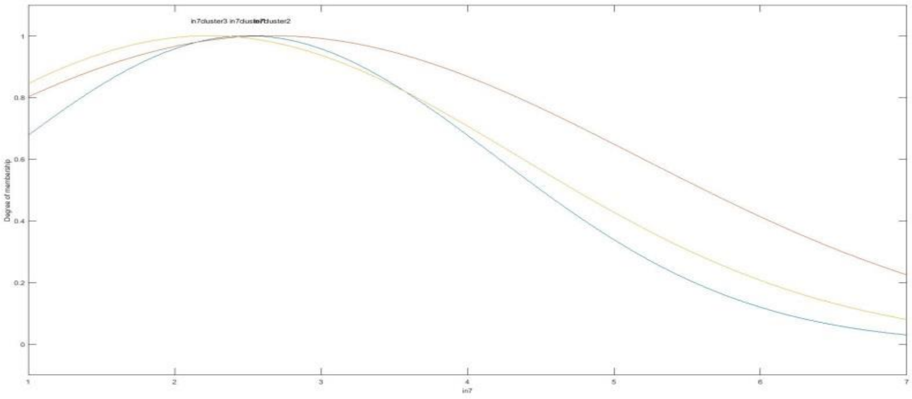
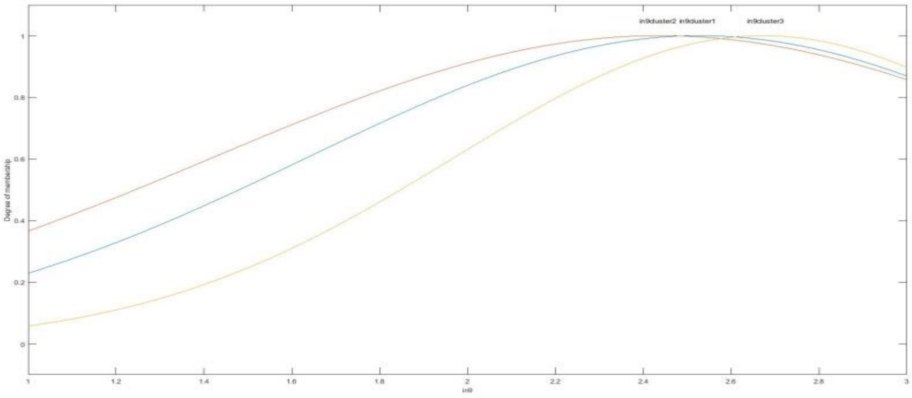

in8cluster3
Degree of membership
in8

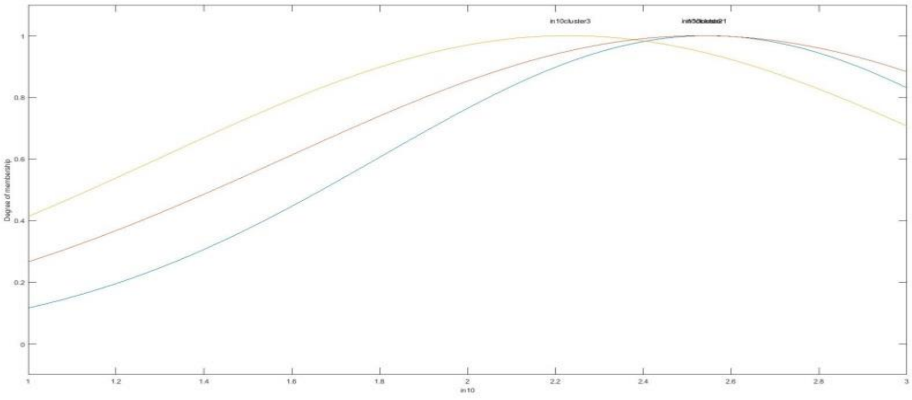
e)


in7cluster3
Degree of membership
in7

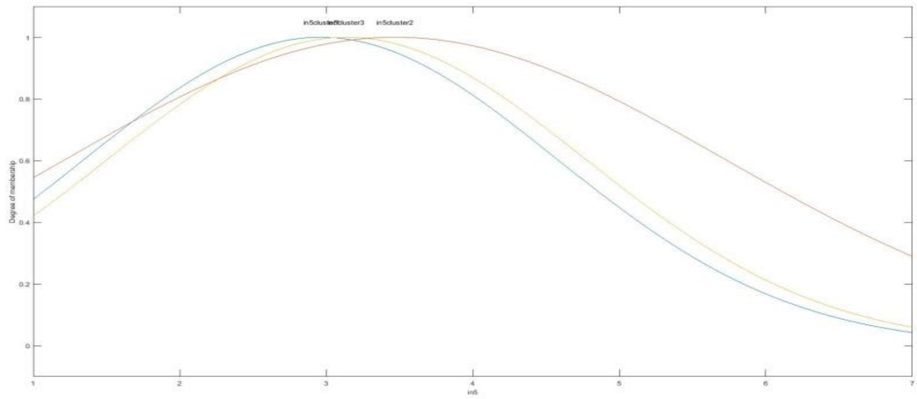
f)


in9cluster2
in9cluster1
in9cluster3
Degree of membership
in9

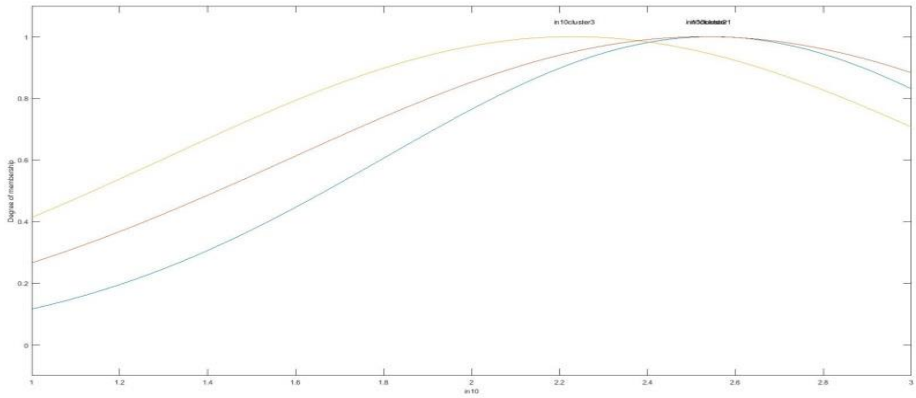
g)


in10cluster3
Degree of membership
in10

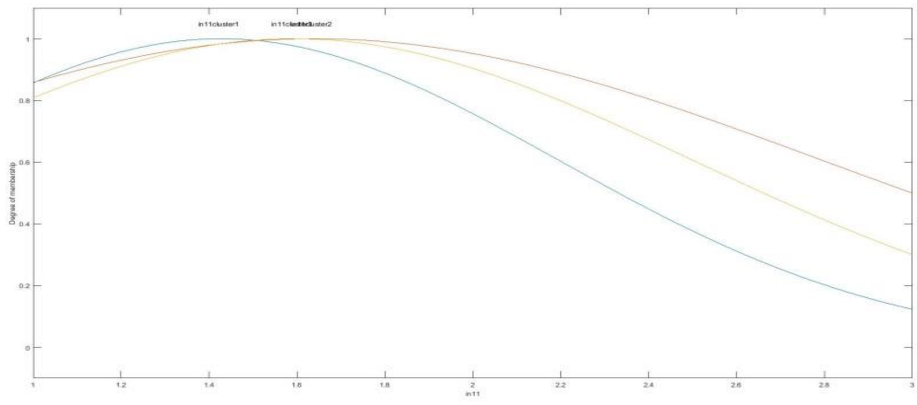
h)



i)



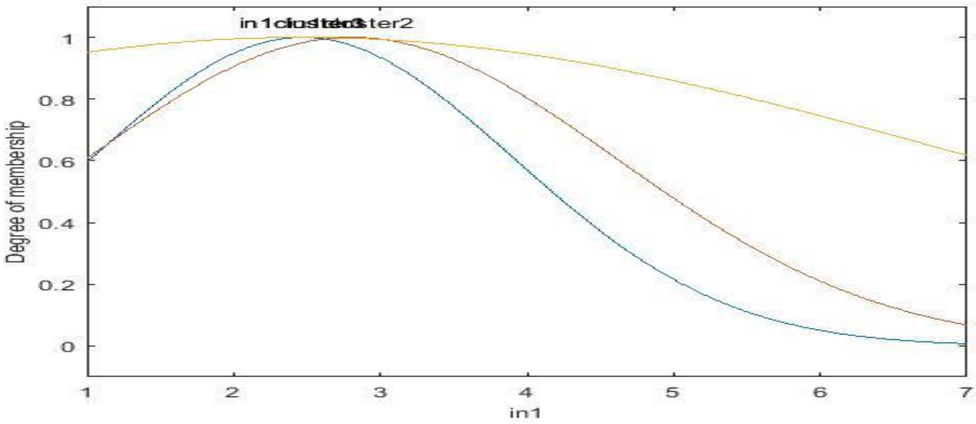
j)



k)

Figure A1.4: Membership functions given by Genfis3 for Job suitability-related Constraints/Facilitators variables: a) Job category b) Time-spent on Reading/Writing c) Time-spent on PC d) Time-spent on Phone/Fax e) Time-spent on Colleagues/Clients f) Time-spent on Attending Meetings/Team Work g) Time-spent on Errands h) Importance of Phone/Fax Machine i) Importance of PC j) Importance of Reports/Correspondents k) Importance of Special places like labs

e) Technology Availability related Constraints/Facilitators variables



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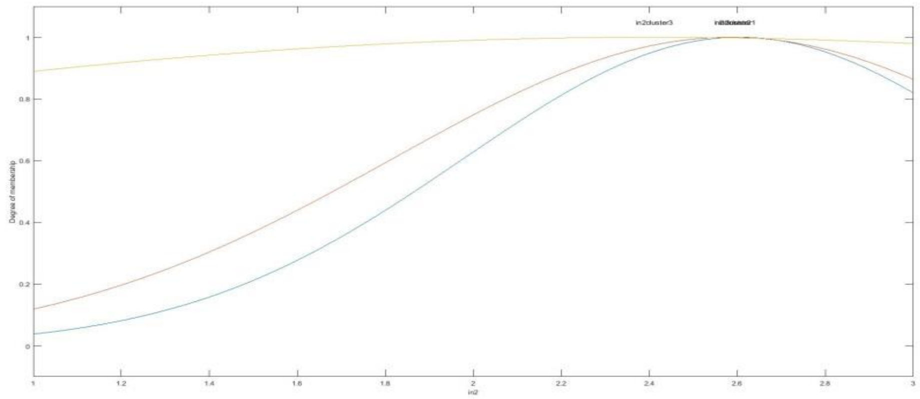

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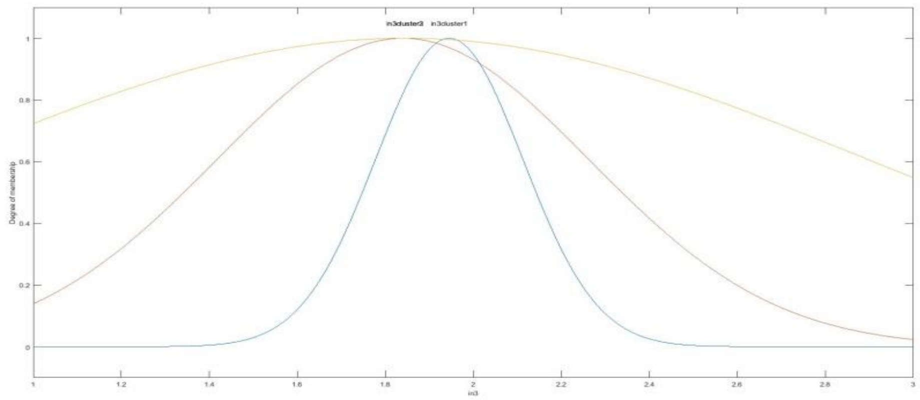
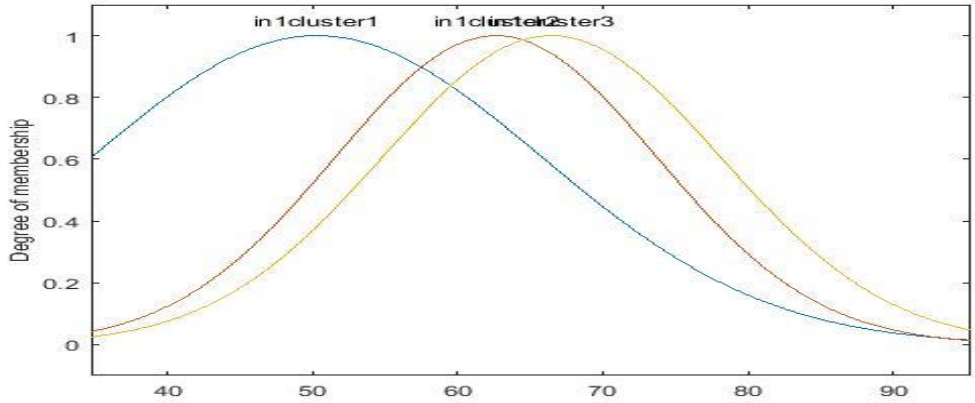


c)

Figure A1.5: Membership functions given by Genfis3 for Technology Availability-related Constraints/Facilitators variables: a) Importance of PC b) Importance of Phone/Fax Machine c) Importance of Photocopier

f) Drivers related variables



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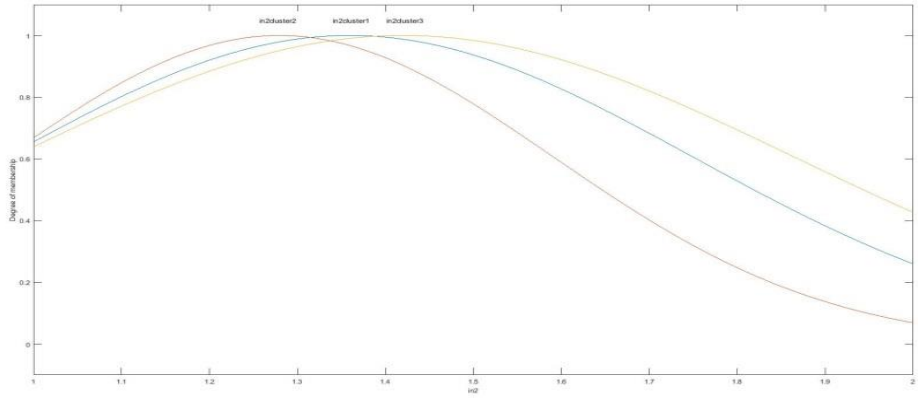

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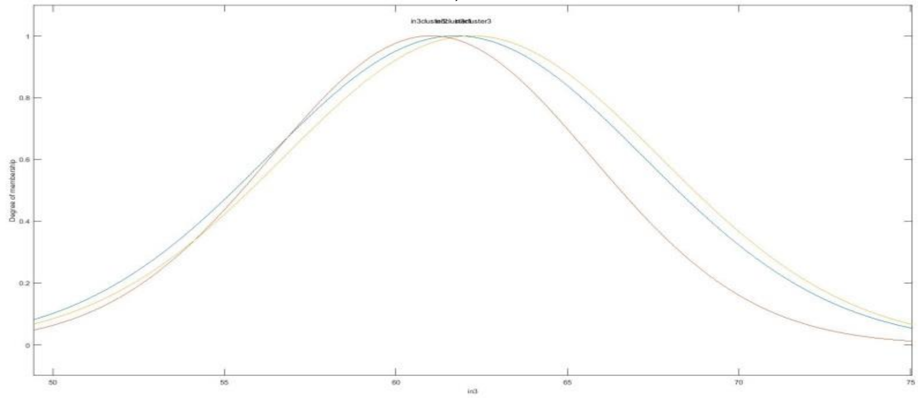

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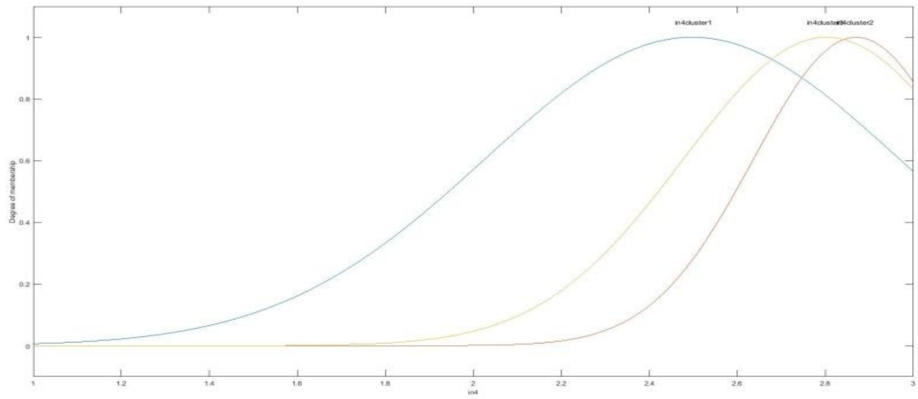
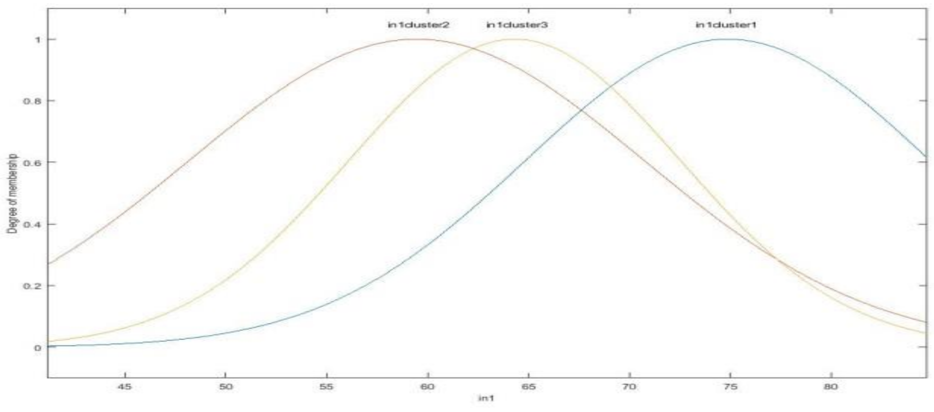


d)

Figure A1.6: Membership functions given by Genfis3 for Drives variables:

a) Work-related drives of PC b) Family-related drives c) Commuting-related drives d) Ideology-related drives

g) Constraints/Facilitators related variables



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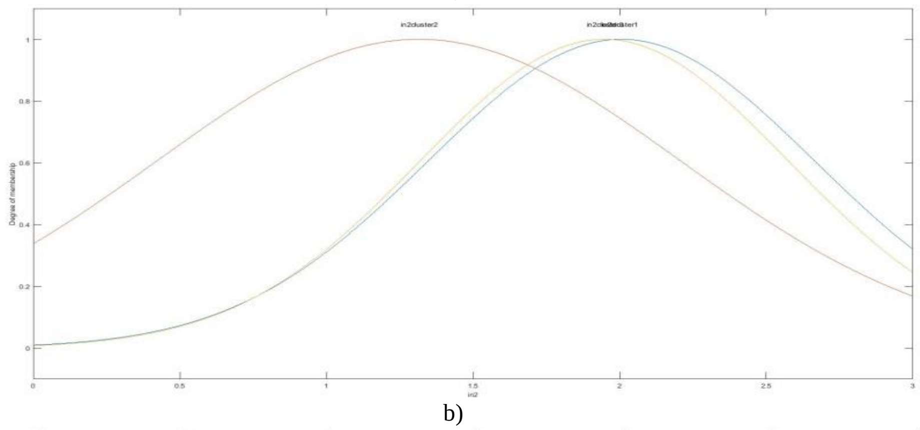

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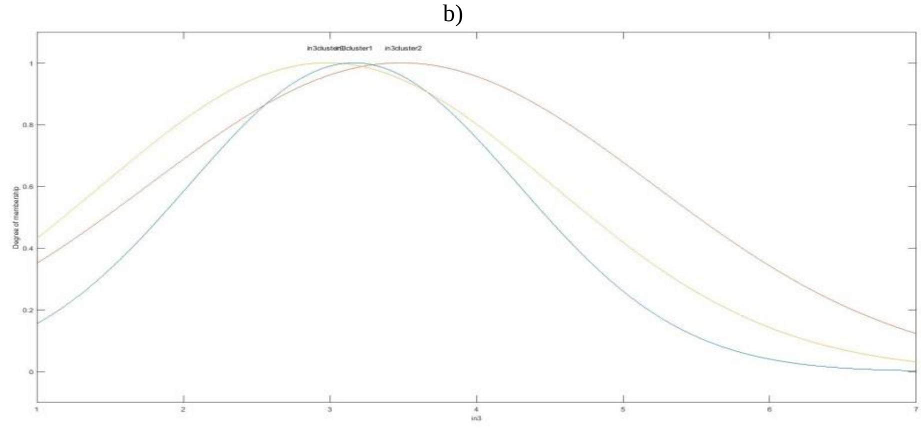

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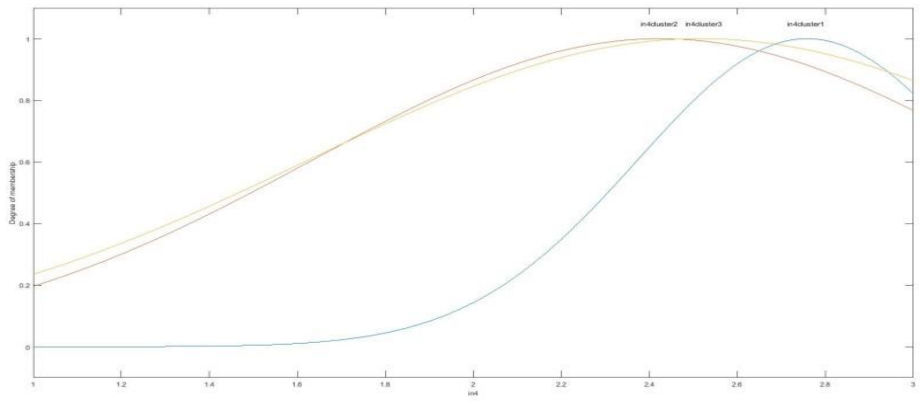
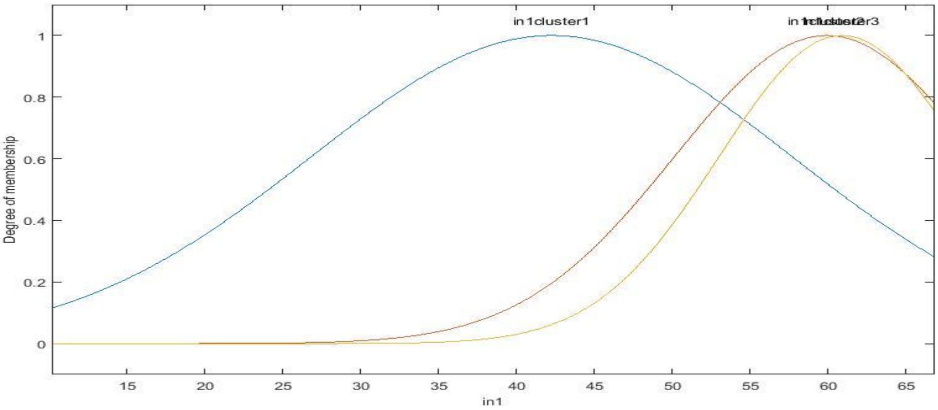


d)

Figure A1.7: Membership functions given by Genfis3 for Constraints/Facilitators variables:

a) Job Suitability b) Manager/Organisation Support c) Social/Professional Interaction d) Technology Availability

h) Suitability of for 0– day telecommuting related variables



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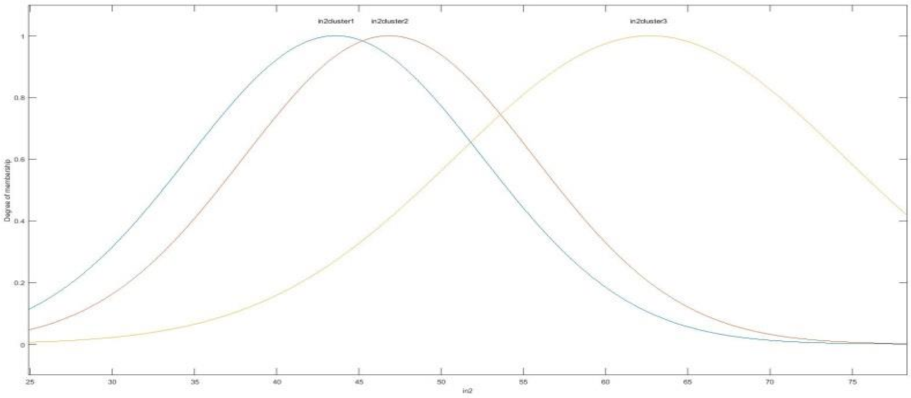

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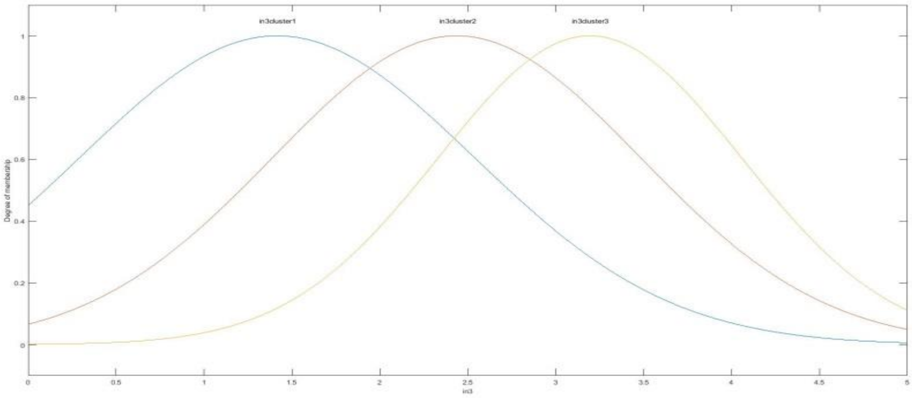

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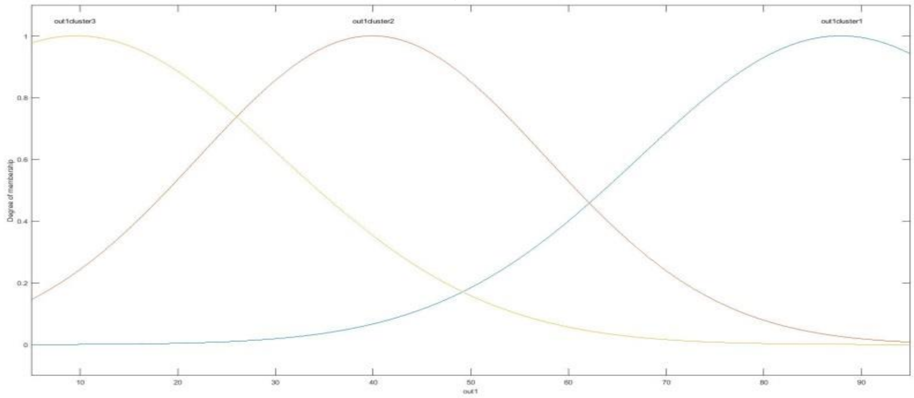
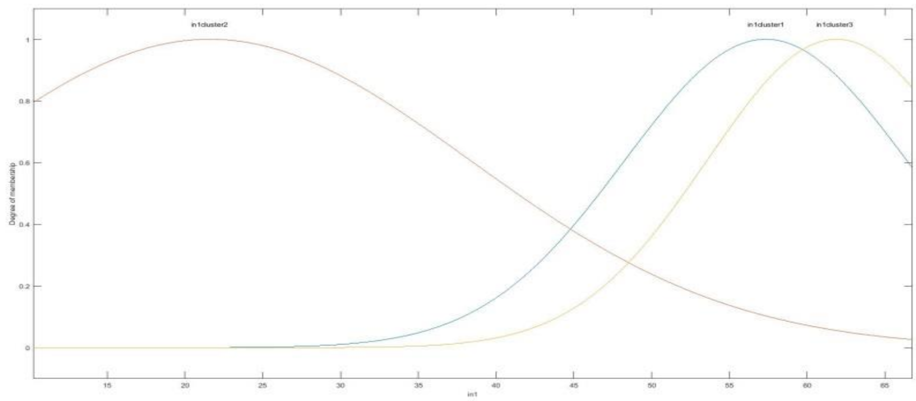


d)

Figure A1.8: Membership functions given by Genfis3 for Suitability of Telecommuting for 0 day a) Drives b) Constraints/Facilitators c) Preference d) Suitability of Telecommuting for 0 day

i) Suitability of for 1–day telecommuting related variables



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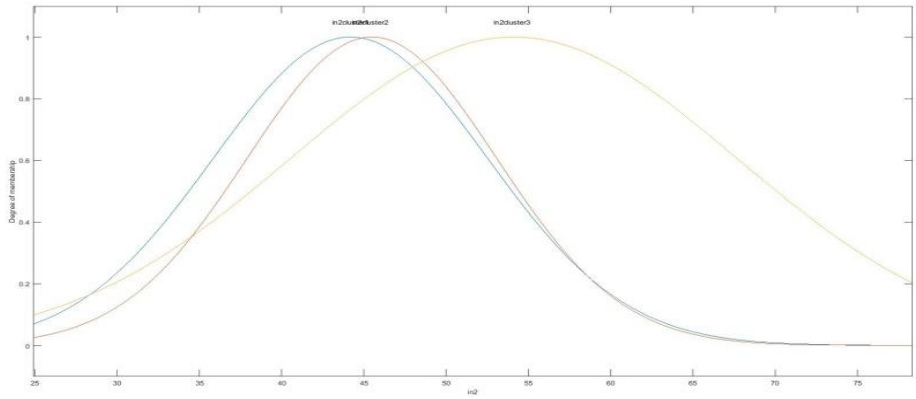

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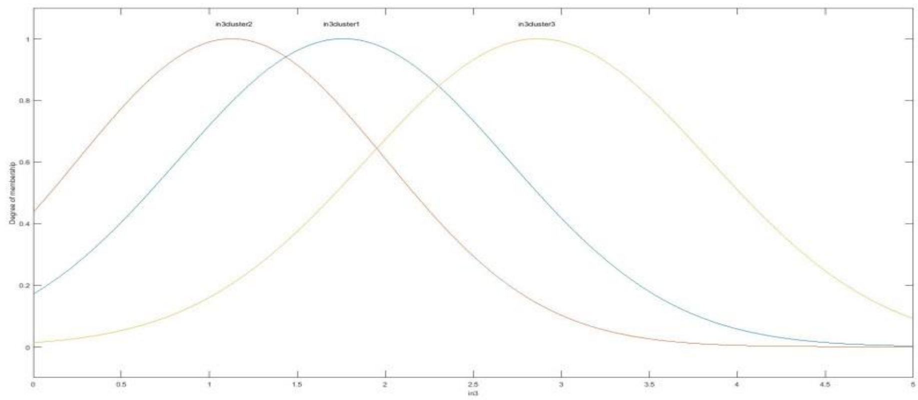

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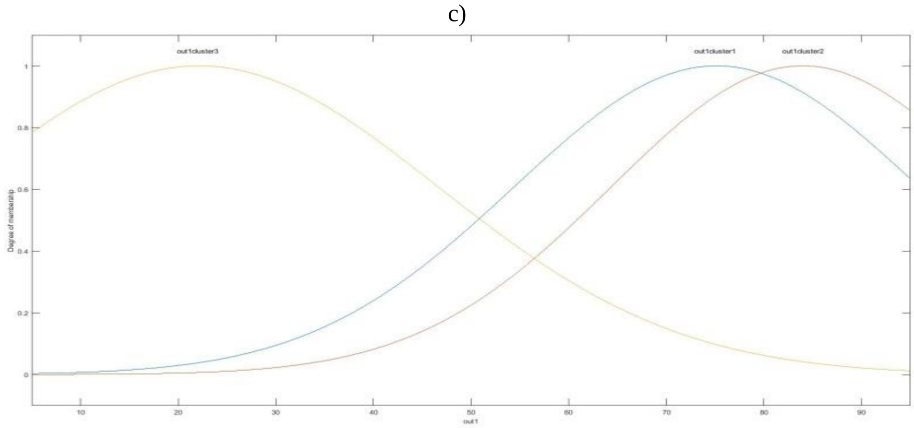
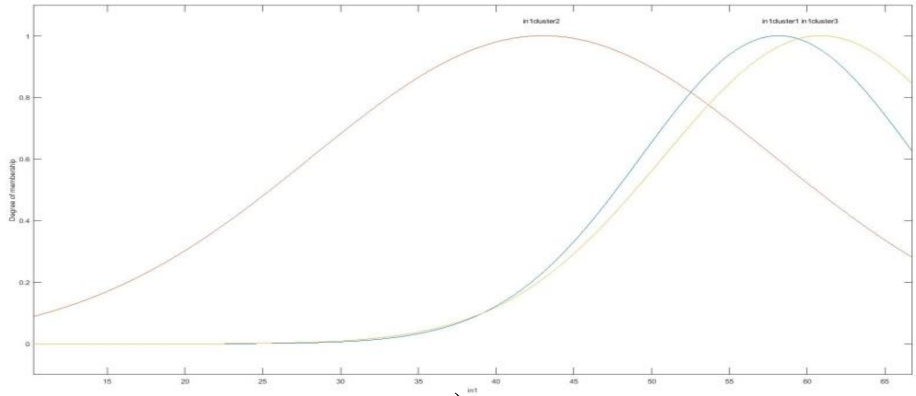


d)

Figure A1.9: Membership functions given by Genfis3 for Suitability of Telecommuting for 1 day a) Drives b) Constraints/Facilitators c) Preference d) Suitability of Telecommuting for 1 day

j) Suitability of for 2–days telecommuting related variables



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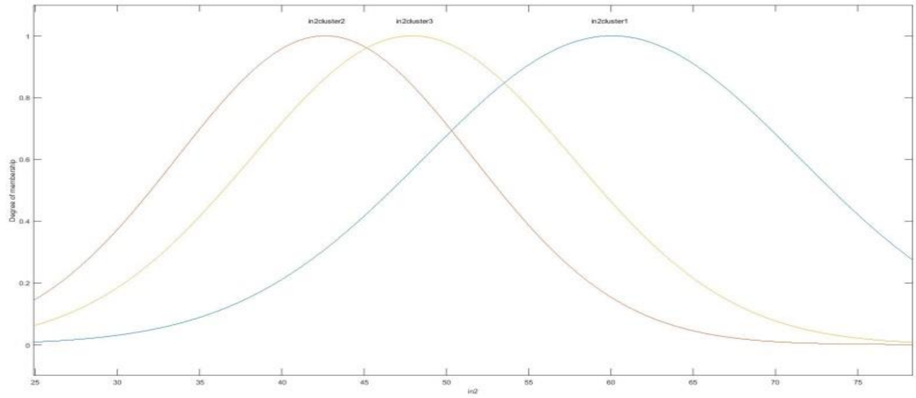

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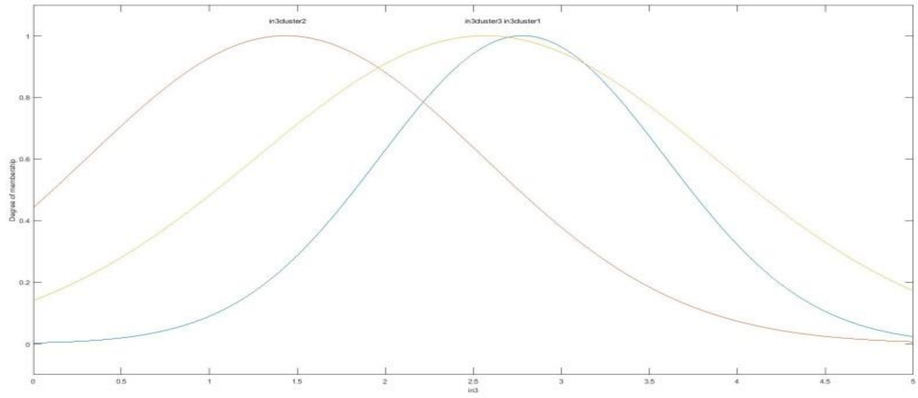

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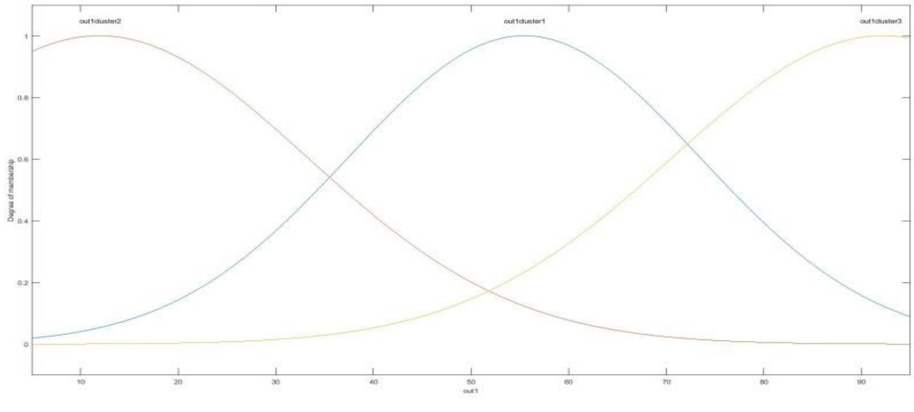
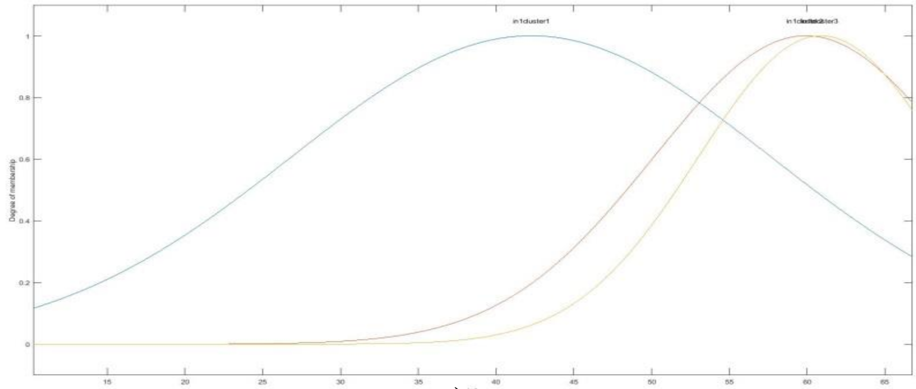


d)

Figure A1.10: Membership functions given by Genfis3 for Suitability of Telecommuting for 2 days a) Drives b) Constraints/Facilitators c) Preference d) Suitability of Telecommuting for 2 days

k) Suitability of for 3–days telecommuting related variables



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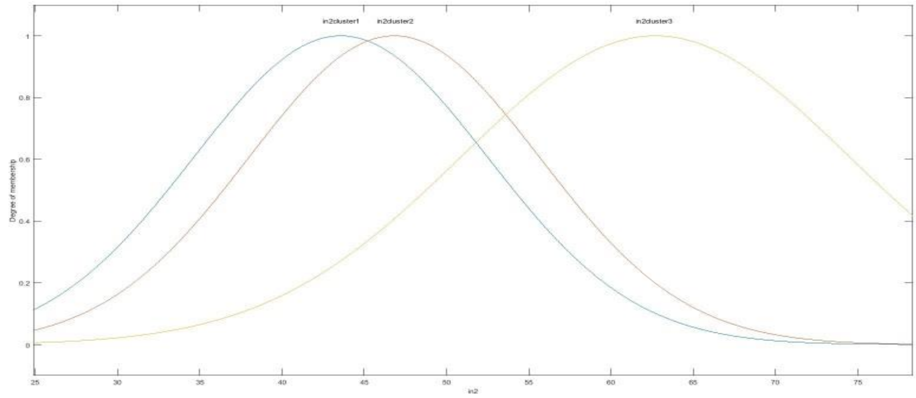

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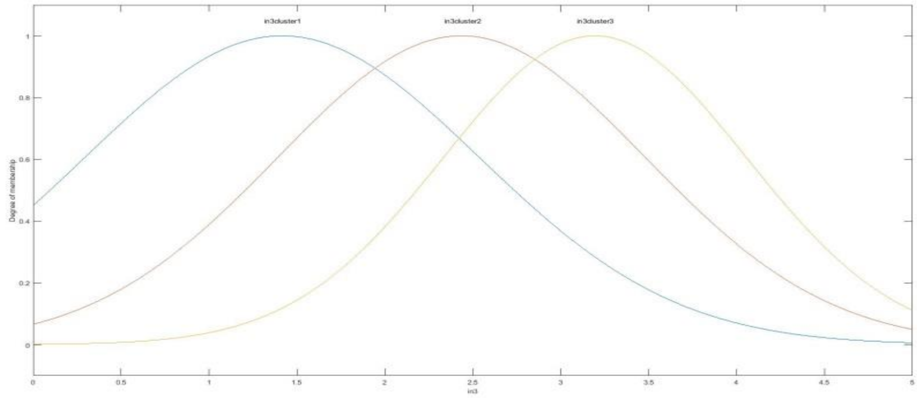

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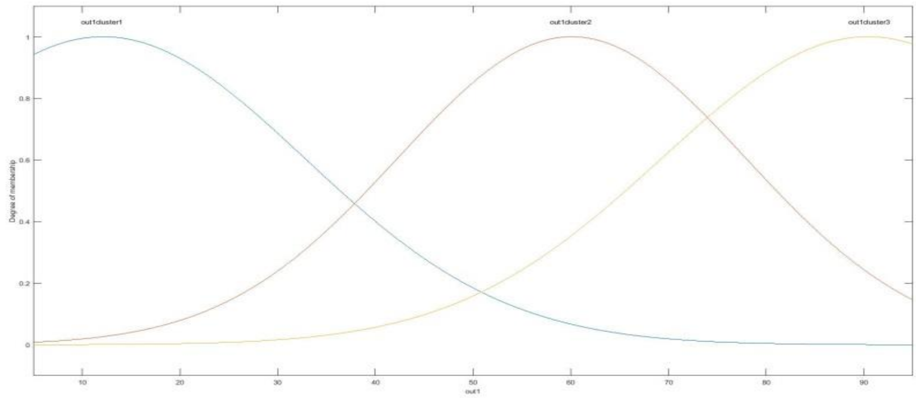


d)

Figure A1.11: Membership functions given by Genfis3 for Suitability of Telecommuting for 3 days a) Drives b) Constraints/Facilitators c) Preference d) Suitability of Telecommuting for 3 days

Appendix 2

Genetic Algorithm minimises the objective functions which is to make observed sample most likely to have occurred. Following figures show the number of iterations for GAs to tune membership functions for different variables in the model.

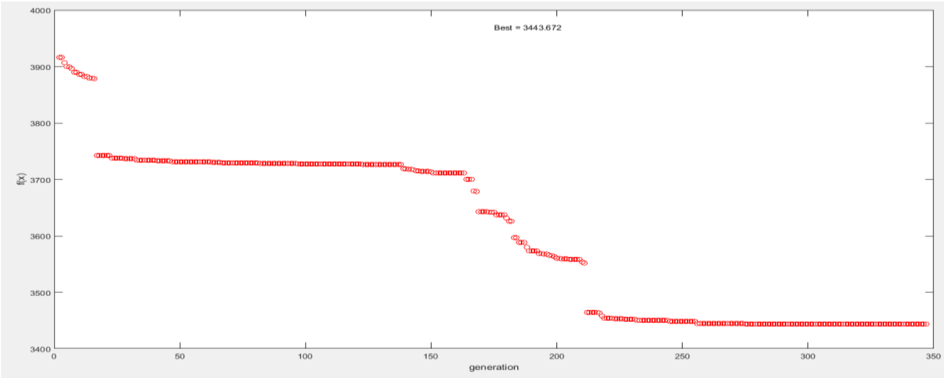


Figure A2.1: GA minimisation for subsystem Family related drive

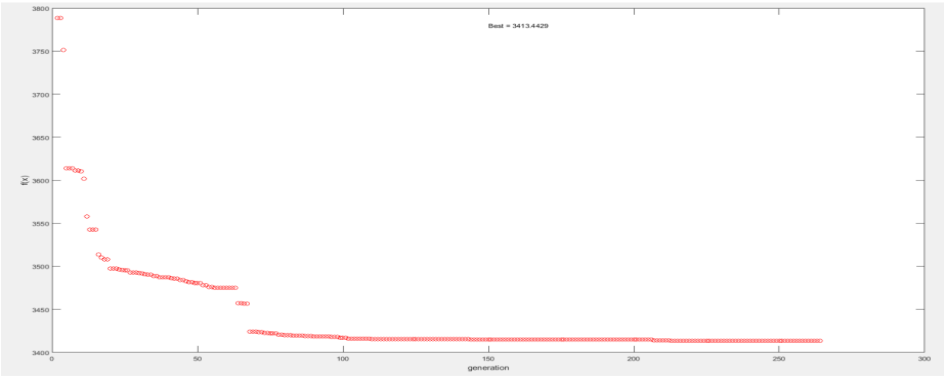


Figure A2.2: GA minimisation for subsystem commuting related drives

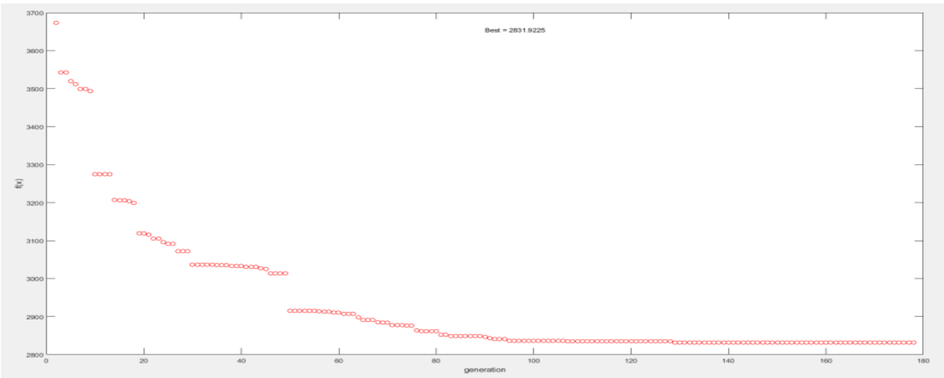


Figure A2.3: GA minimisation for subsystem Ideology related drives

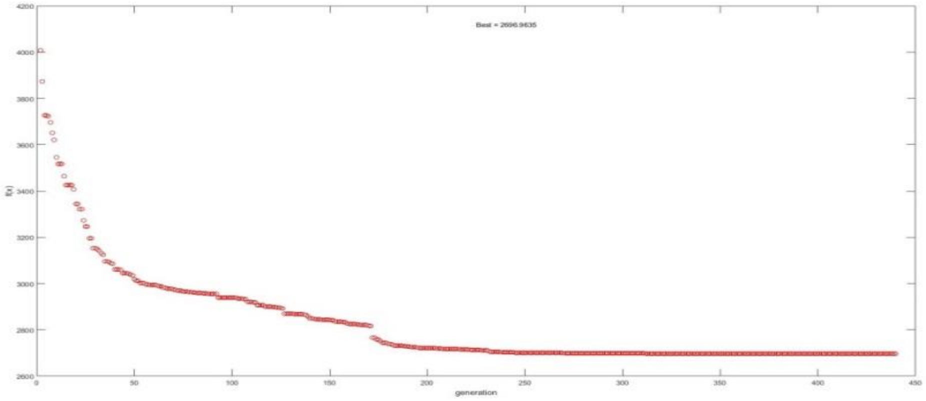


Figure A2.4: GA minimisation for subsystem Job suitability-related related Constraints/Facilitators

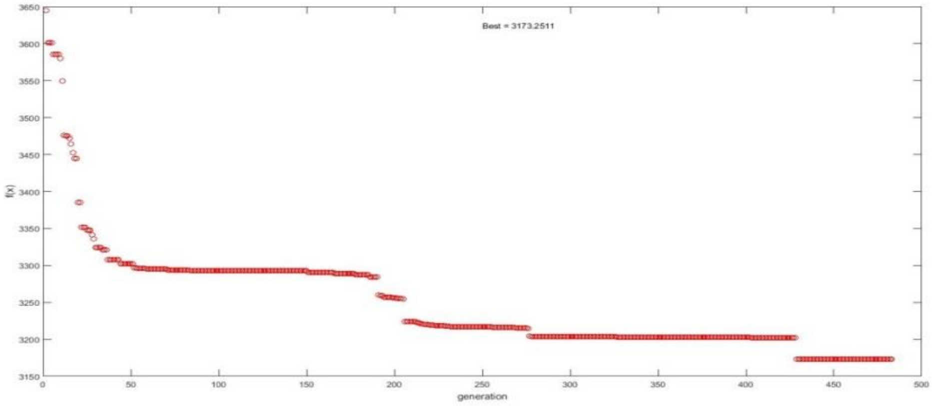


Figure A2.5: GA minimisation for subsystem Technology Availability-related Constraints/Facilitators

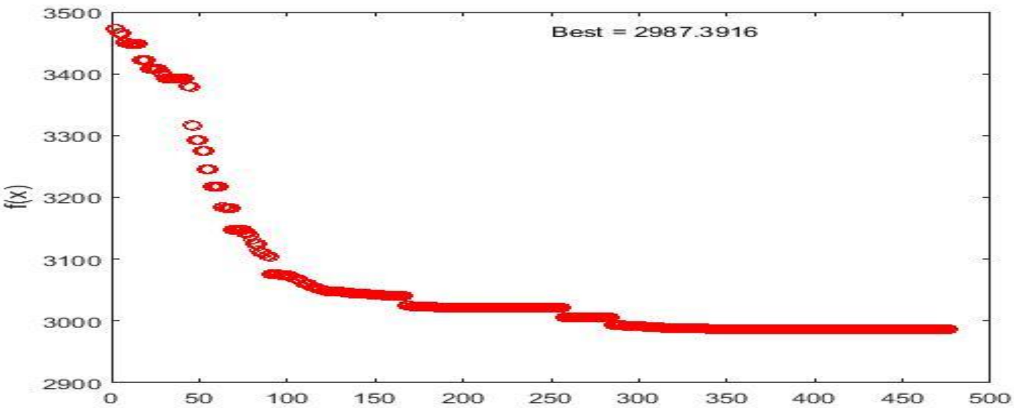


Figure A2.6: GA minimisation for subsystem Drives

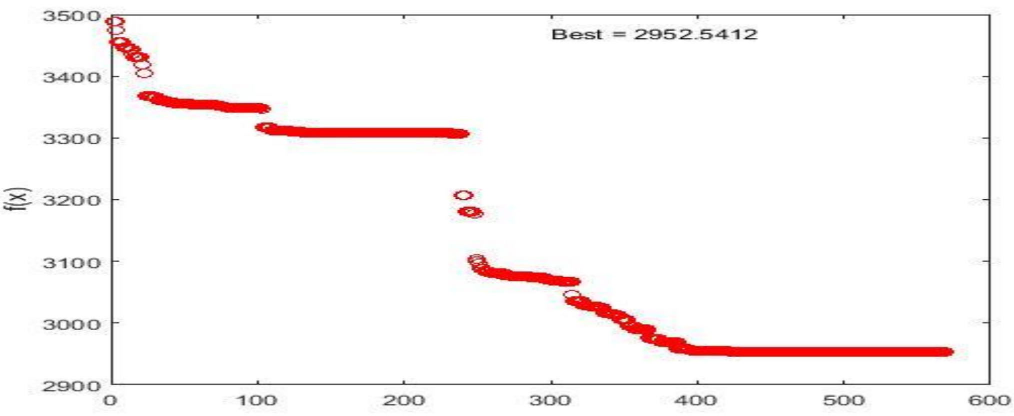


Figure A2.7: GA minimisation for subsystem Constraints/Facilitators

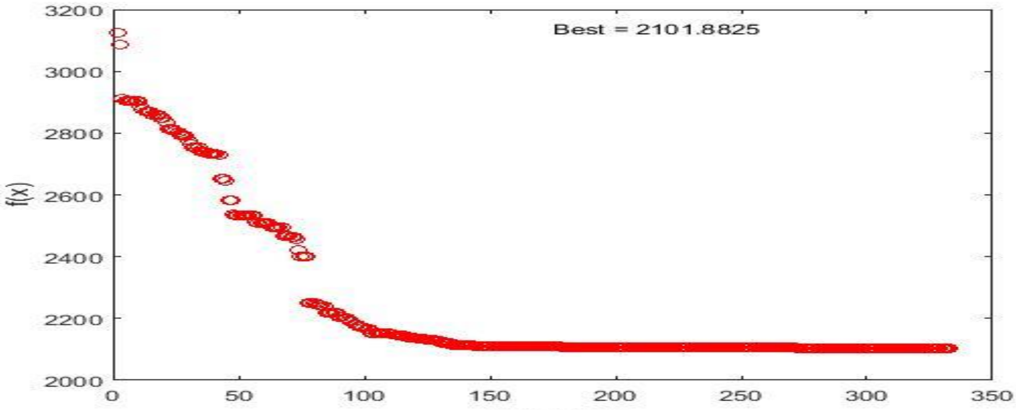


Figure A2.8: GA minimisation for suitability of Telecommuting for 0 day

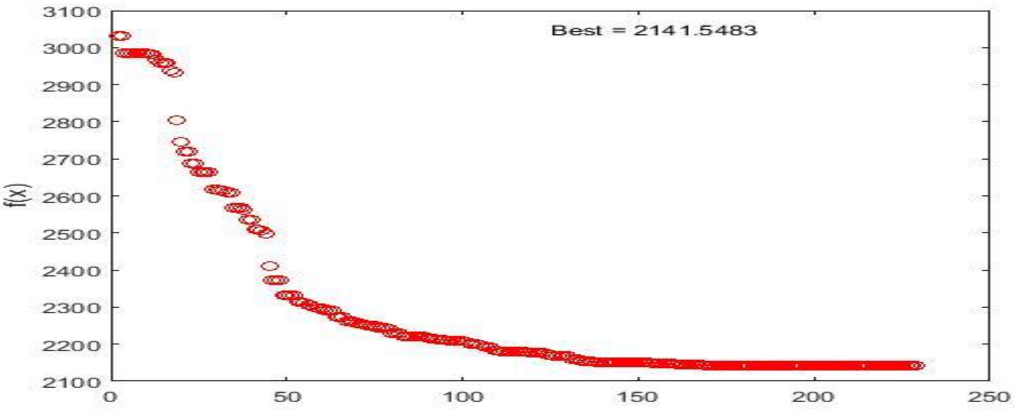


Figure A2.9: GA minimisation for suitability of Telecommuting for 1 day

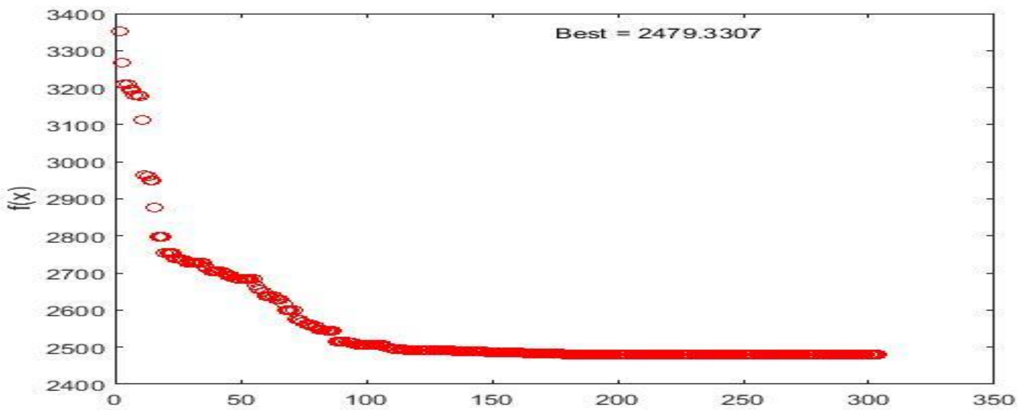


Figure A2.10: GA minimisation for suitability of Telecommuting for 2 days

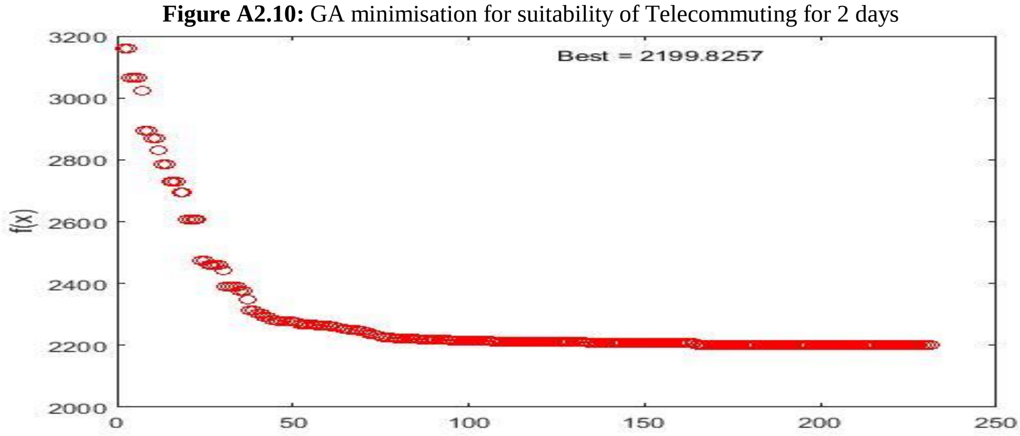
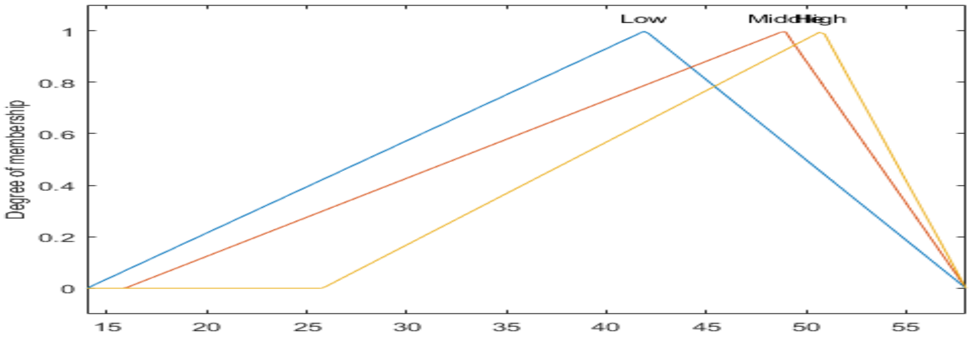


Figure A2.11: GA minimisation for suitability of Telecommuting for 3 days

Appendix 3

Tunned membership functions obtained from Genetic Algorithm optimisation process.

a) Family related drive variables



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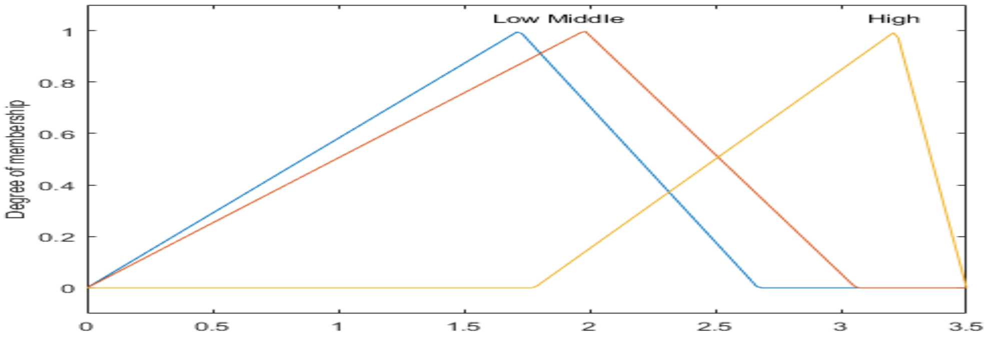

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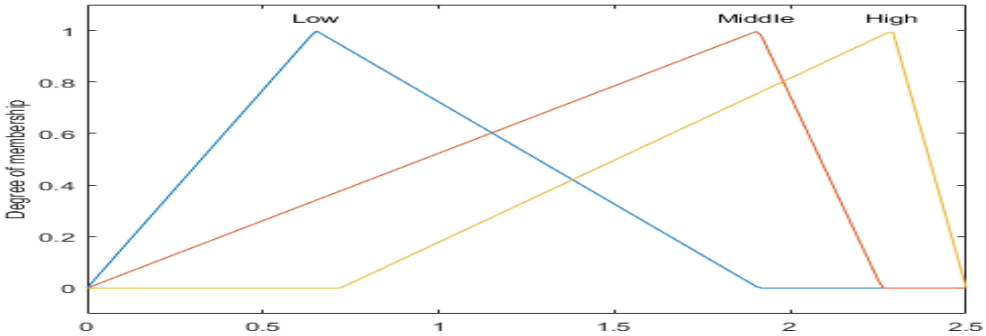

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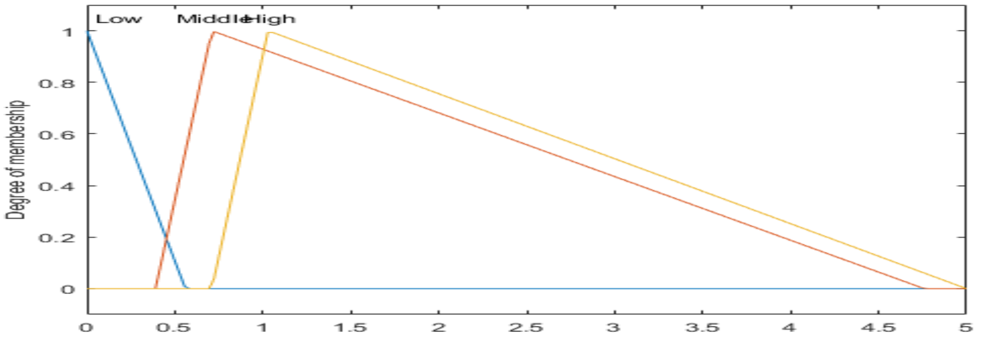
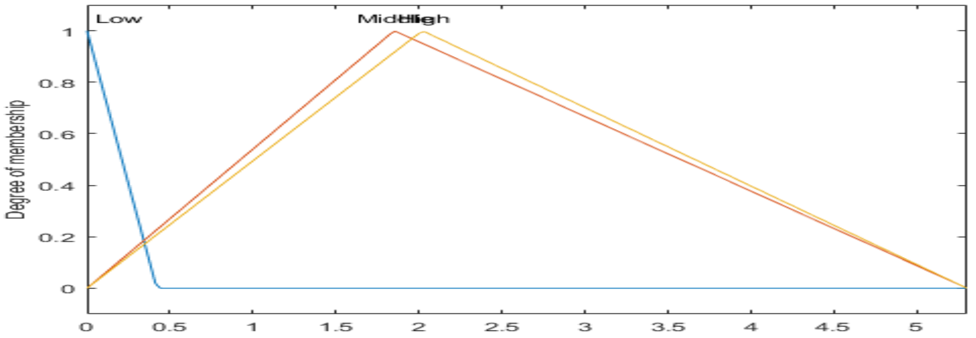


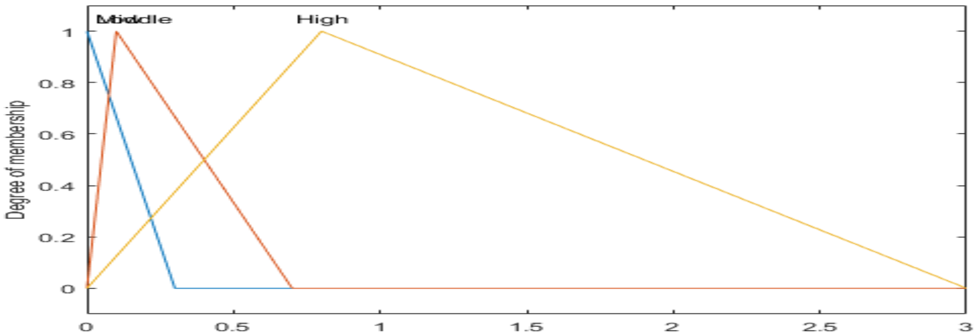
d)



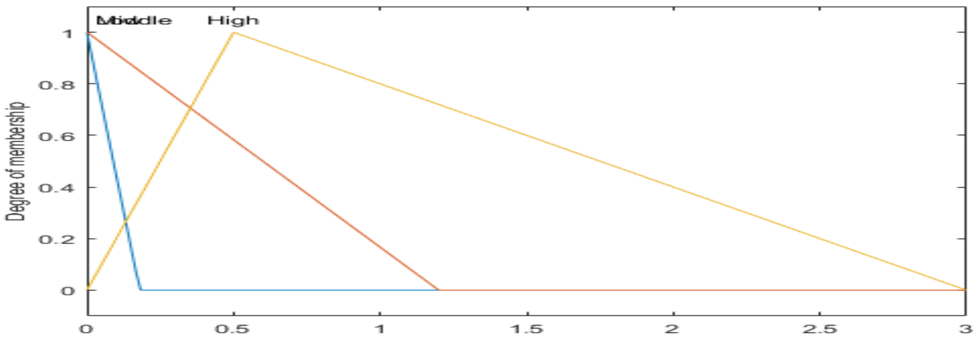
e)

Figure A3.1: Optimised membership functions for family related drive variables: a) Age b) Gender c) Marital Status d) Number of children e) Family wellbeing

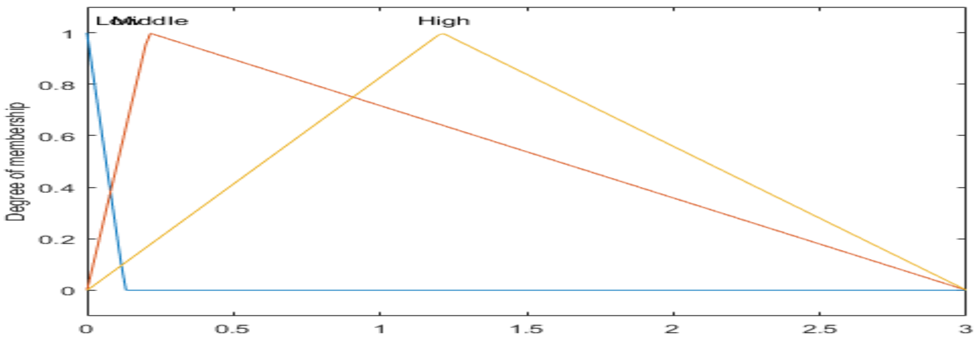
b) Commuting related drive variables



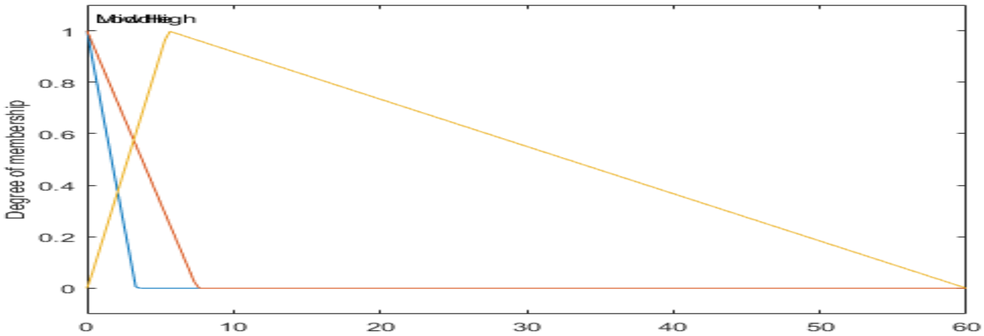
a)



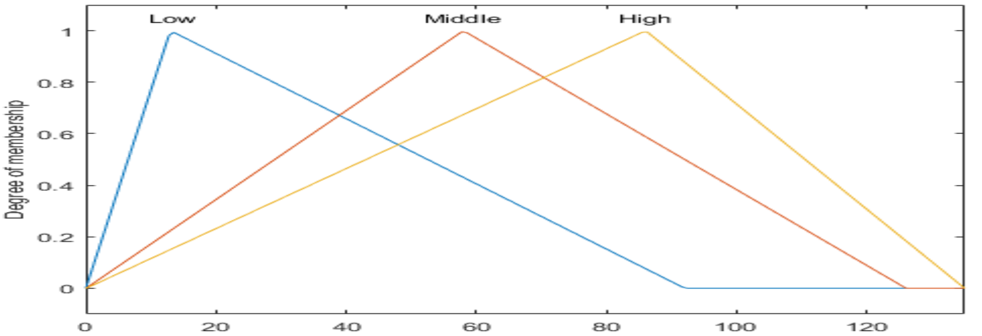
b)



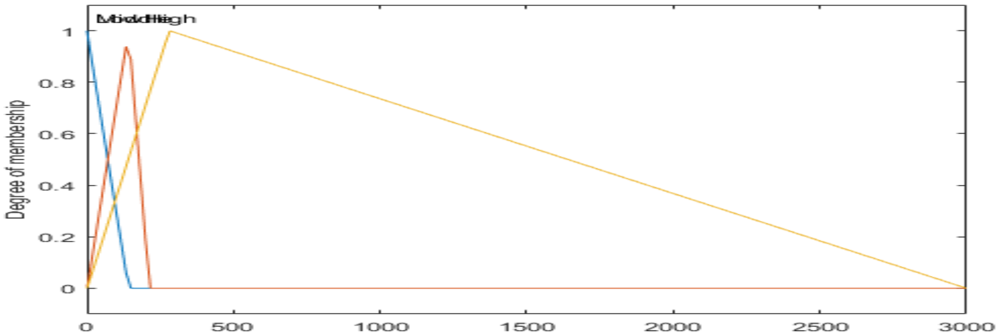
c)



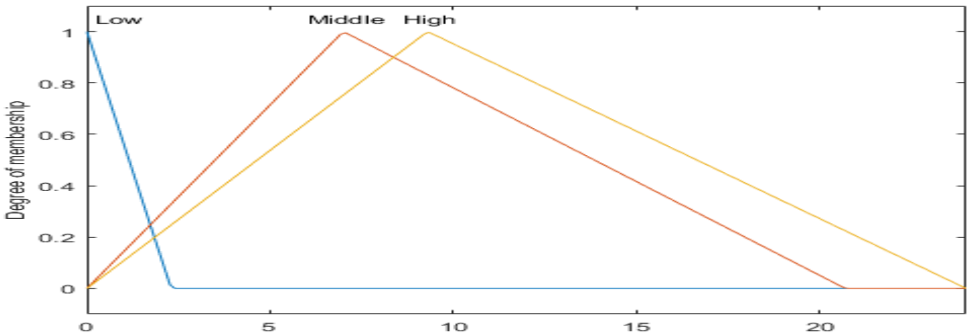
d)



e)



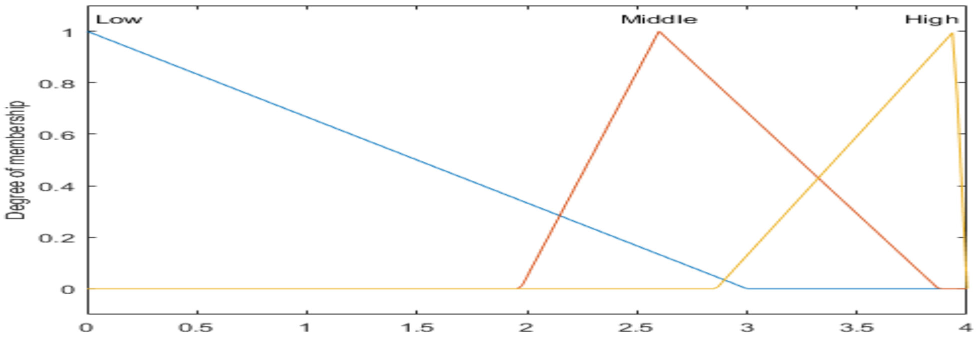
f)



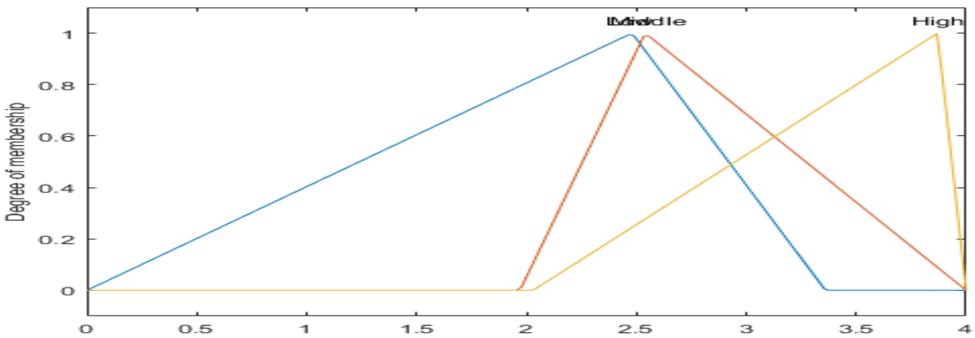
g)

Figure A3.2: Optimised membership functions for Commuting related drives variables:
a) Holding driving license b) Car ownership c) Using organisation's car d) Commuting distance e)Travel cost f)Travel time g)Ideology on influence of telecommuting on congestion reduction

c) Ideology related drive variables



a)



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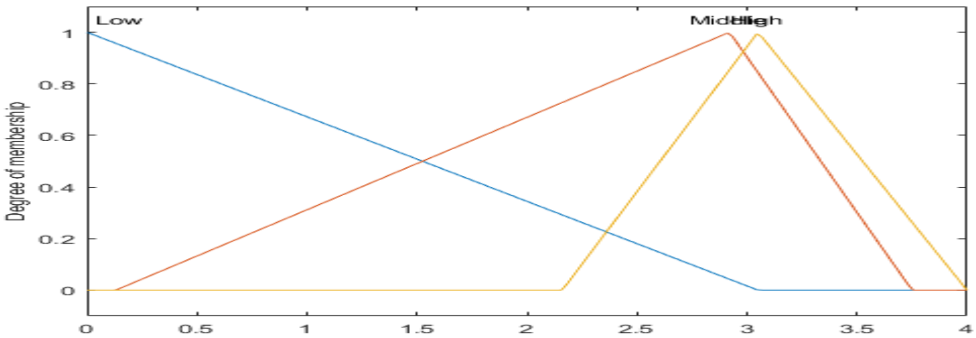
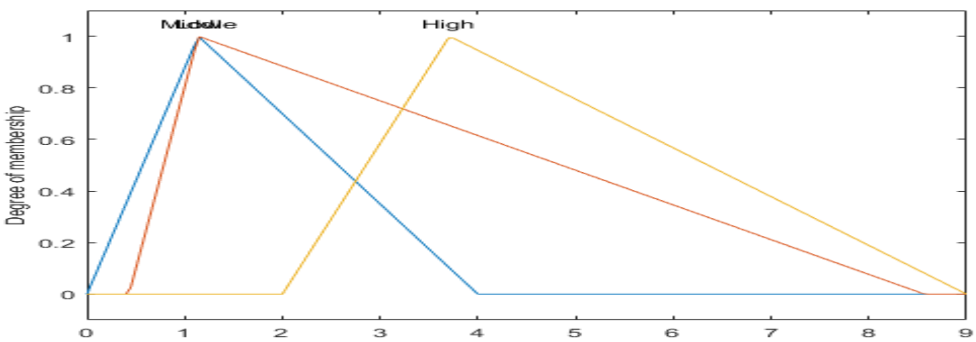


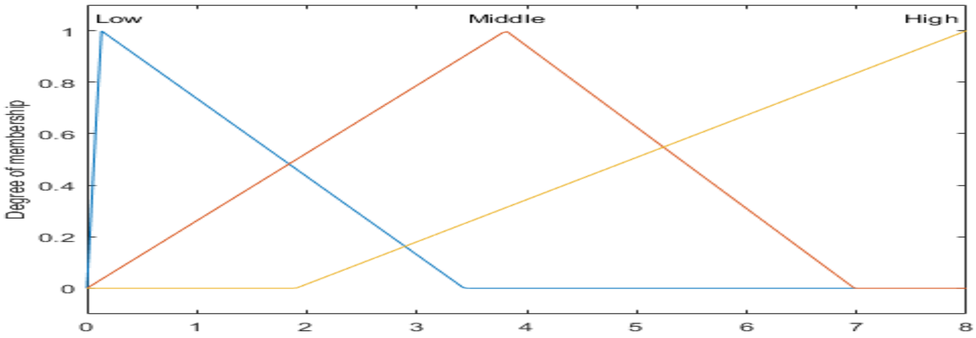
c)

Figure A3.3: Optimised membership functions for Ideology related drives variables:
a) Family wellbeing b) Productivity in job c) Congestion reduction

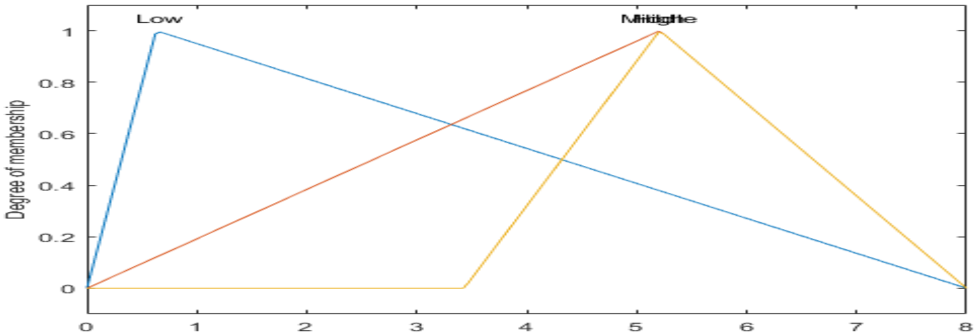
d) Job suitability related Constraints/Facilitators variables



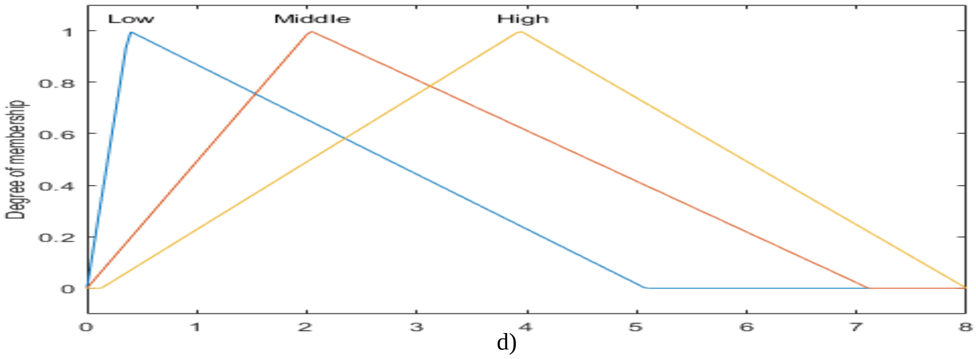
a)


Low
Middle
High
Degree of membership
1
0.8
0.6
0.4
0.2
0
0 1 2 3 4 5 6 7 8

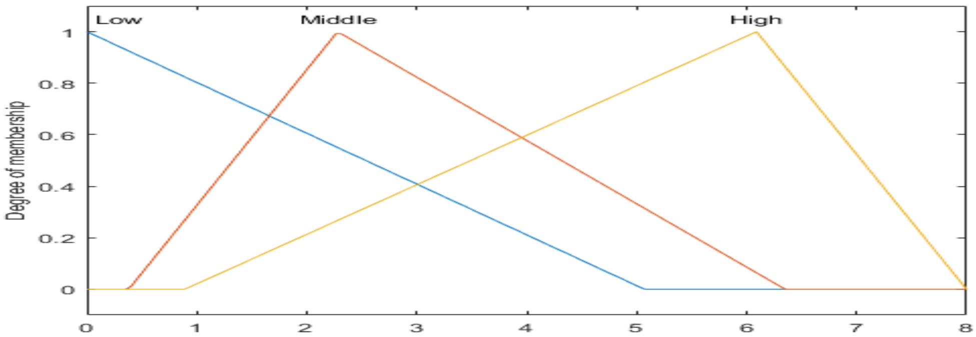
b)


Low
Degree of membership
1
0.8
0.6
0.4
0.2
0
0 1 2 3 4 5 6 7 8

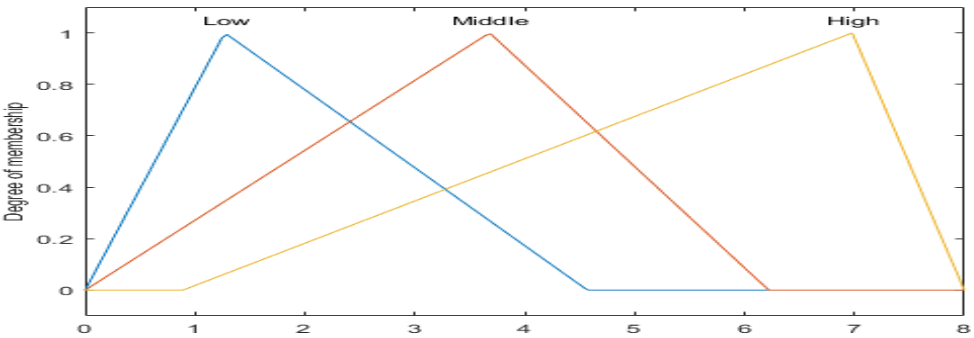
c)


Low
Middle
High
Degree of membership
1
0.8
0.6
0.4
0.2
0
0 1 2 3 4 5 6 7 8

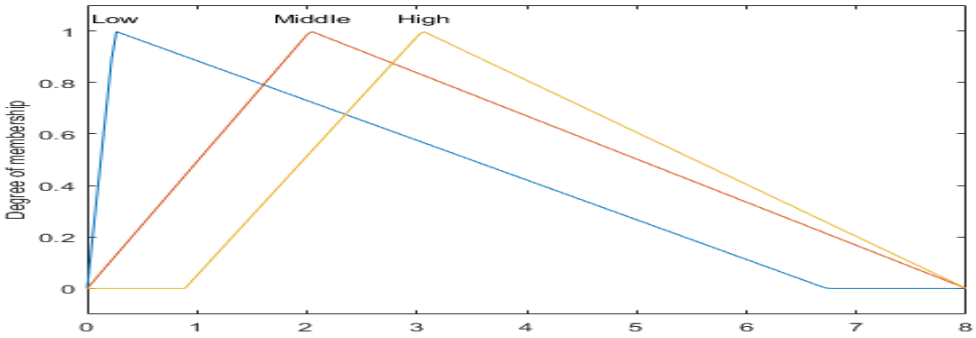
d)


Low
Middle
High
Degree of membership
1
0.8
0.6
0.4
0.2
0
0 1 2 3 4 5 6 7 8

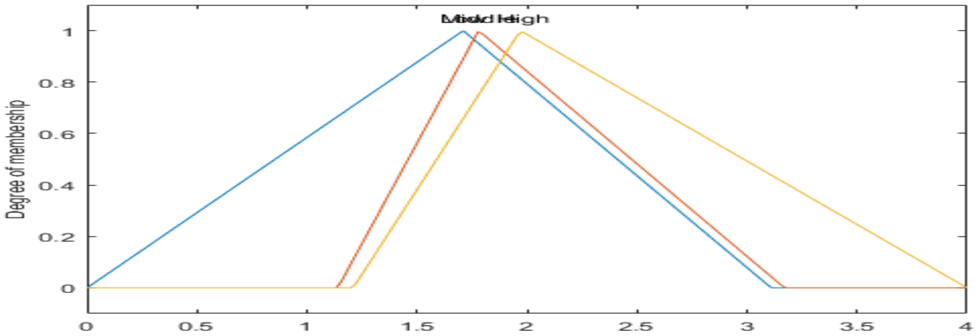
e)


Low
Middle
High
Degree of membership
1
0.8
0.6
0.4
0.2
0
0 1 2 3 4 5 6 7 8

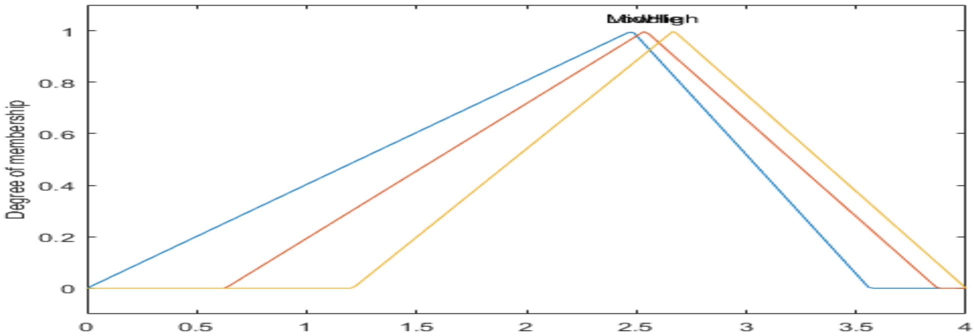
f)


Low
Middle
High
Degree of membership

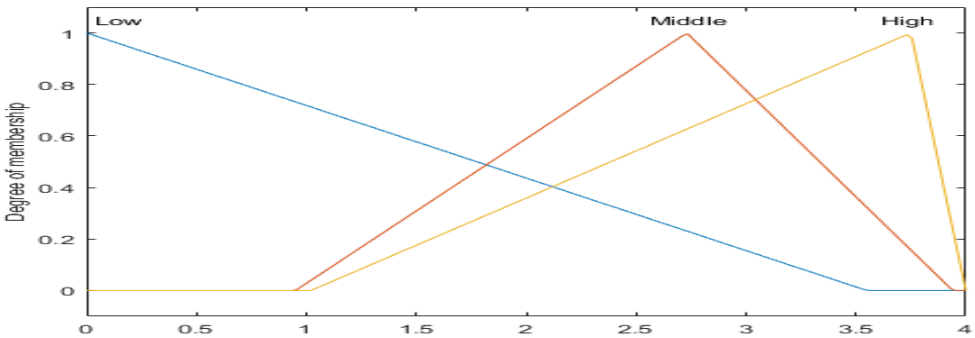
g)


Degree of membership

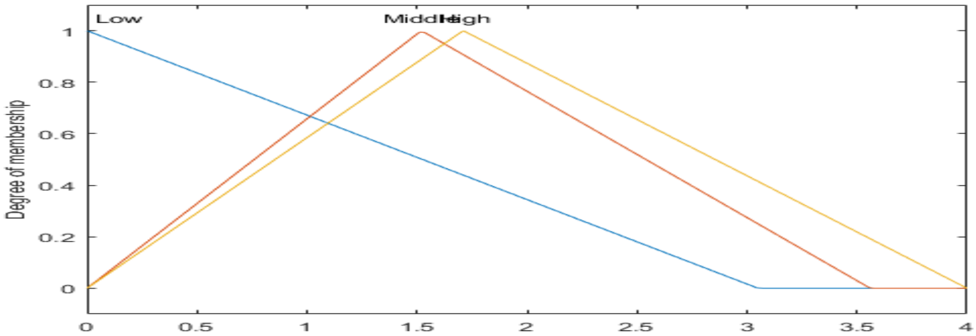
h)


Degree of membership

i)


Low
Middle
High
Degree of membership

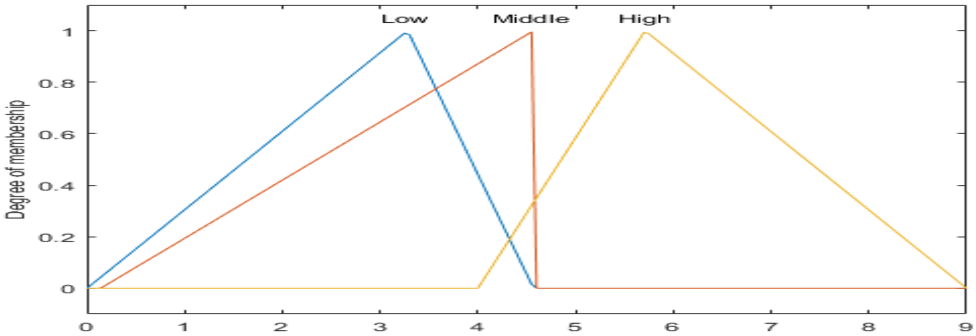
j)



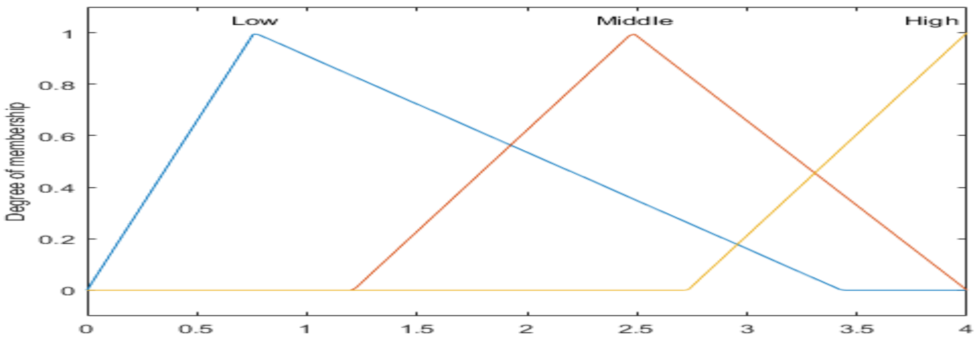
k)

Figure A3.4: Optimised membership functions for Job suitability-related related Constraints/Facilitators variables
a) Job category b) Time-spent on Reading/Writing c) Time-spent on PC d) Time-spent on Phone/Fax e) Time-spent on Colleagues/Clients f) Time-spent on Attending Meetings/Team Work g) Time-spent on Errands h) Importance of Phone/Fax Machine i) Importance of PC j) Importance of Reports/Correspondents k) Importance of Special places like labs

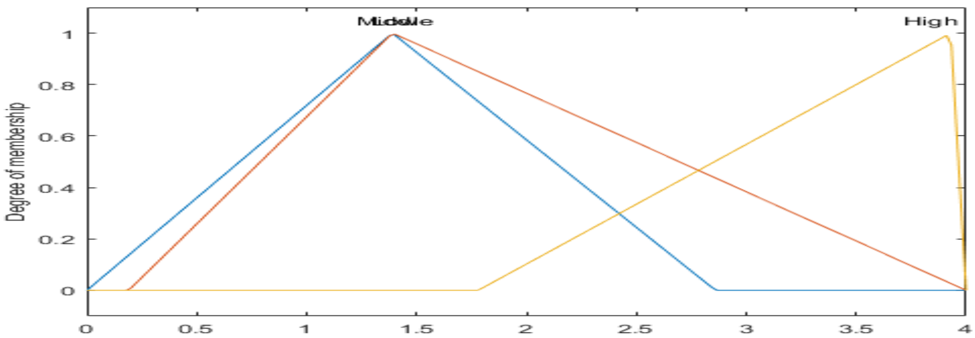
e) Technology Availability related Constraints/Facilitators variables



a)



b)

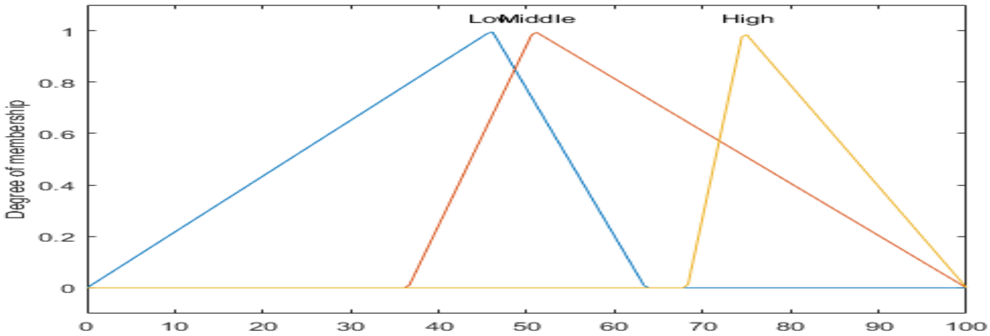


c)

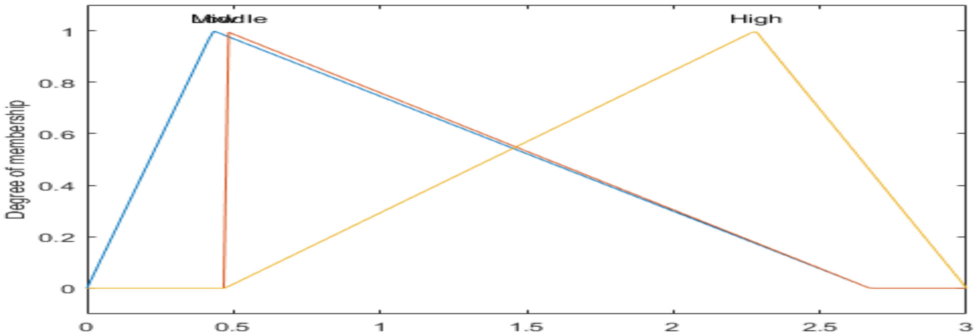
Figure A3.5: Optimised membership functions for Technology Availability-related Constraints/Facilitators variables

a) Importance of PC b) Importance of Phone/Fax Machine c) Importance of Photocopier

f) Drivers related variables



a)



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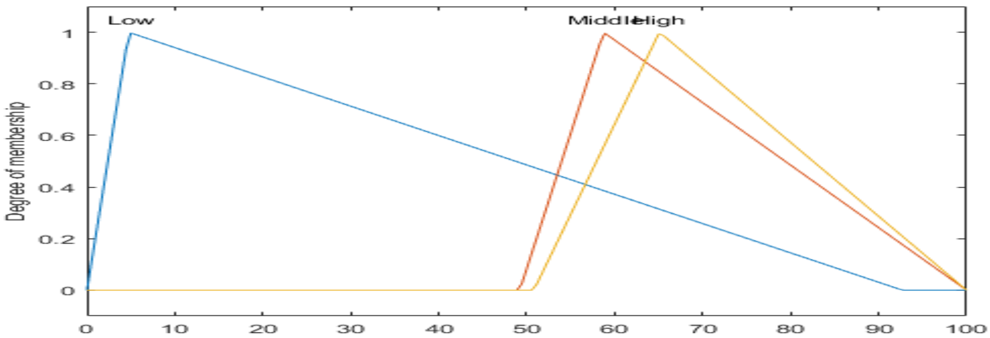

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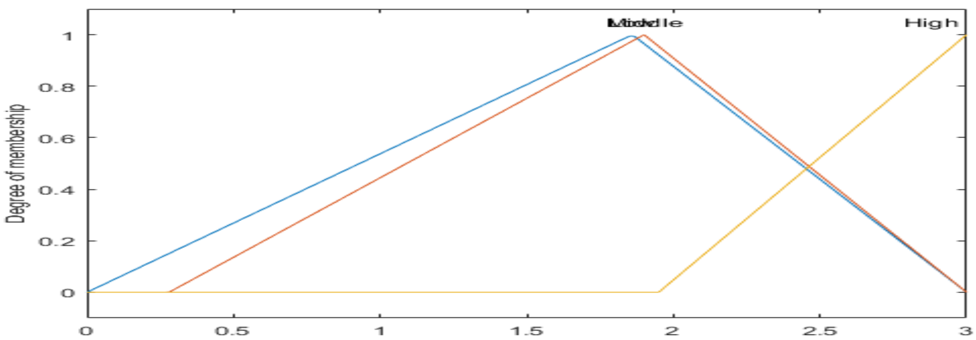
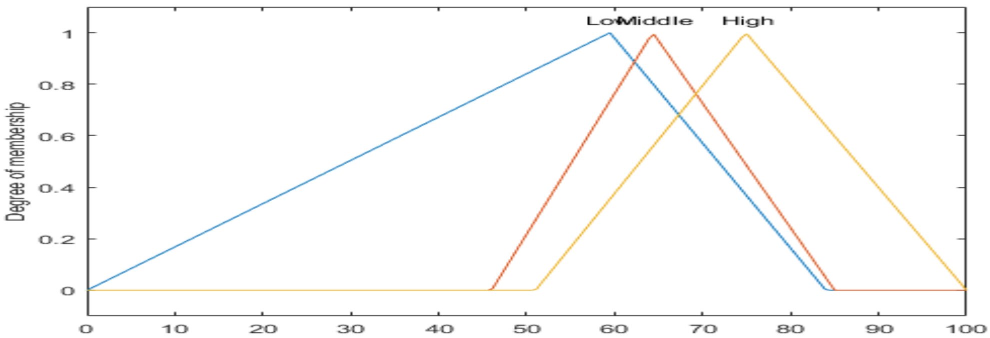


d)

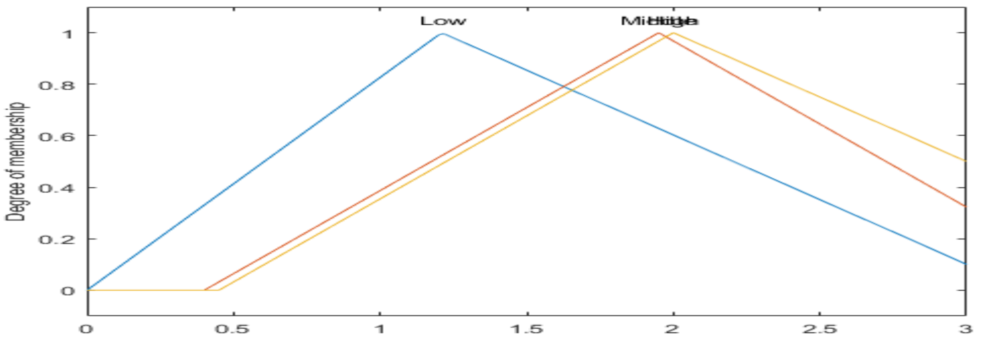
Figure A3.6: Optimised membership functions for Drives variables

a) Work-related drives of PC b) Work-related drives c) Work-related drives d) Work-related drives

g) Constraints/Facilitators related variables



a)



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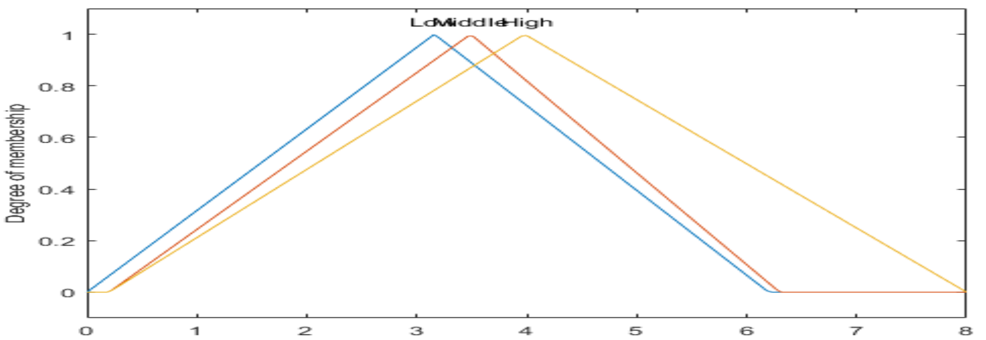

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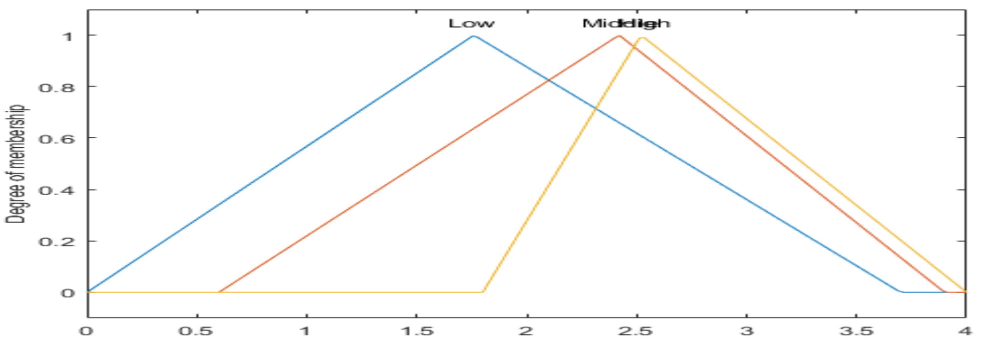


d)

Figure A3.7: Optimised membership functions for Constraints/Facilitators variables:
a) Job Suitability b) Manager/Organisation Support c) Social/Professional Interaction d) Technology Availability

h) Suitability of for 0– day telecommuting related variables

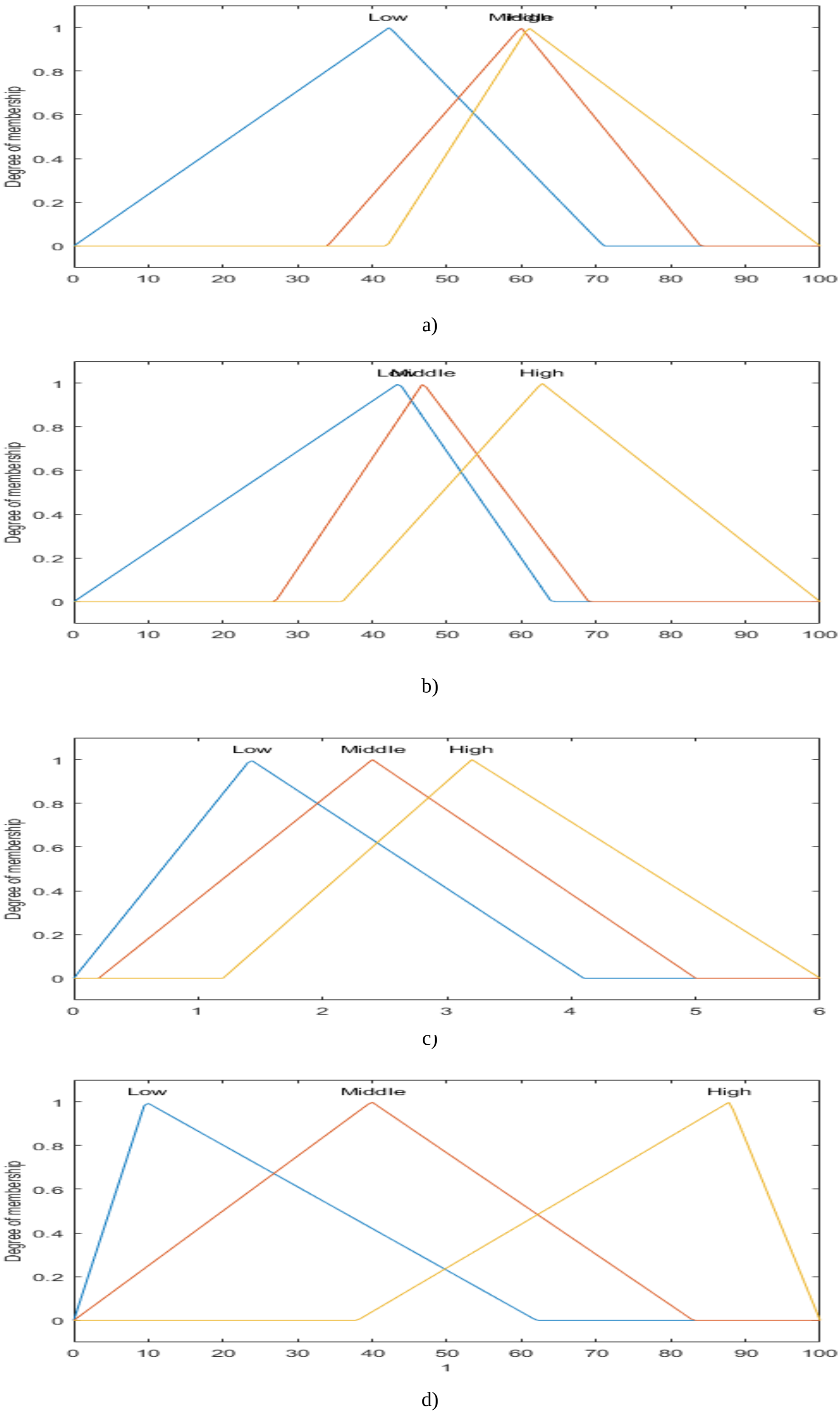
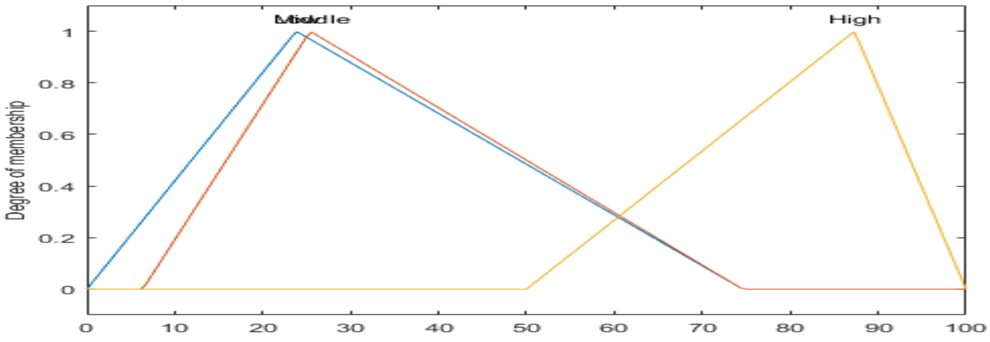
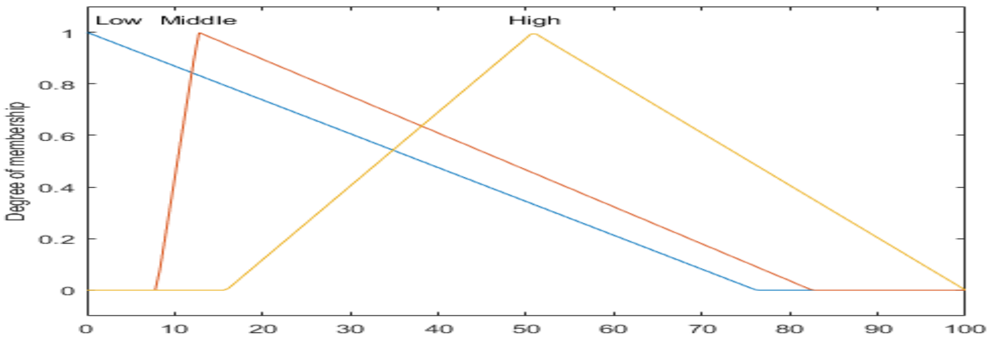


Figure A3.8: Optimised membership functions for Suitability of Telecommuting for 0 day a) Drives b) Constraints/Facilitators c) Preference d) Suitability of Telecommuting for 0 day

i) Suitability of for 1–day telecommuting related variables



a)



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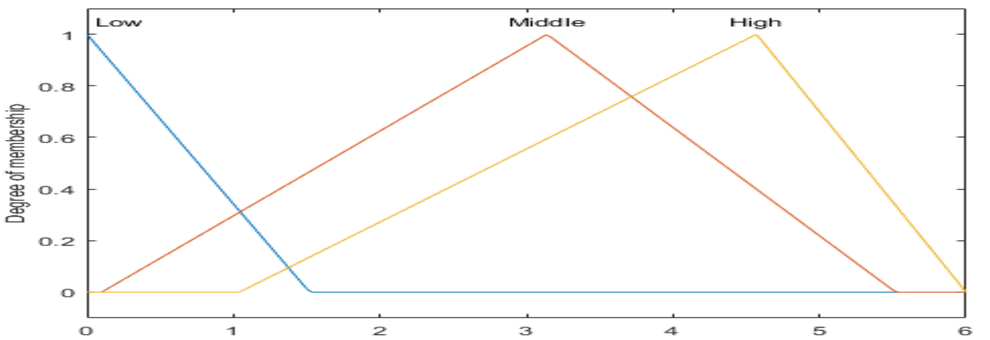
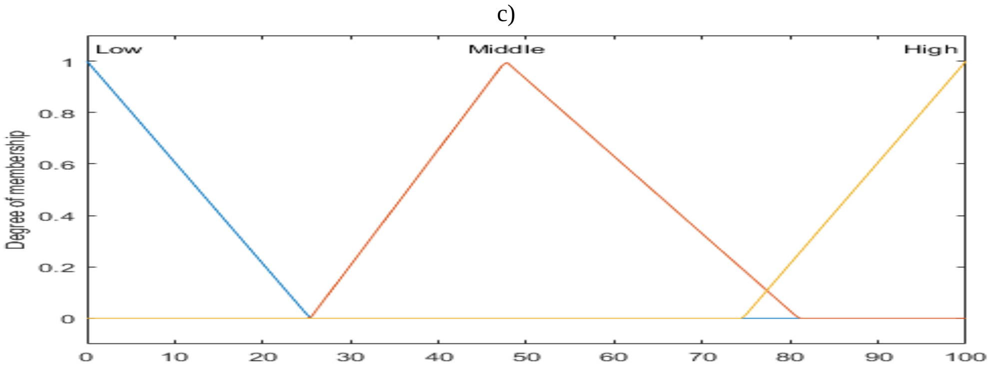


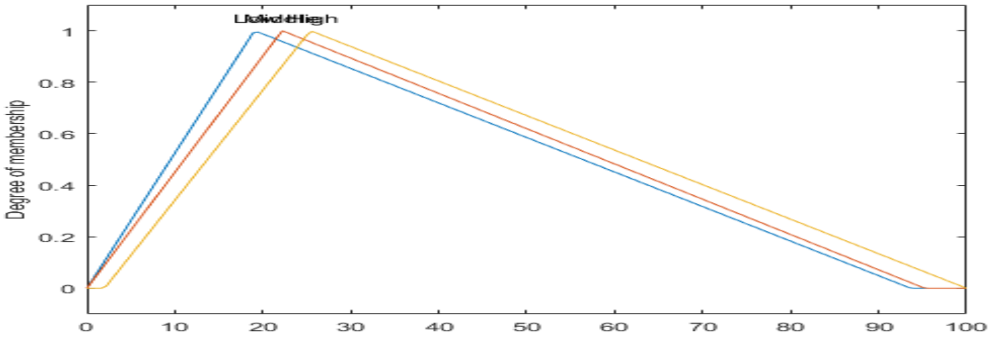
c)



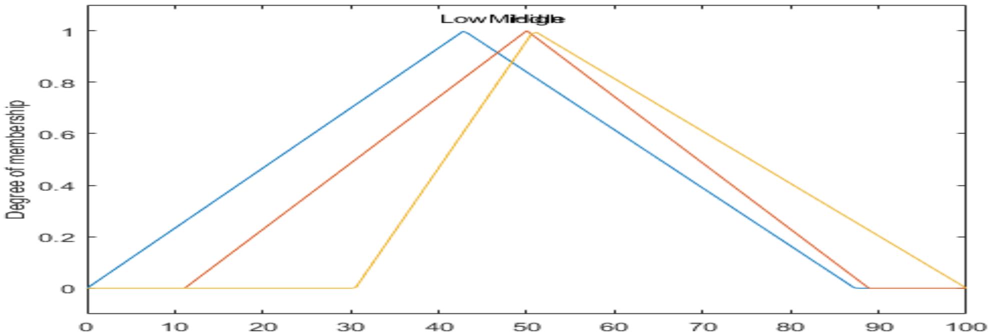
d)

Figure A3.9: Optimised membership functions for Suitability of Telecommuting for 1 day a) Drives b) Constraints/Facilitators c) Preference d) Suitability of Telecommuting for 1 day

j) Suitability of for 2–days telecommuting related variables



a)



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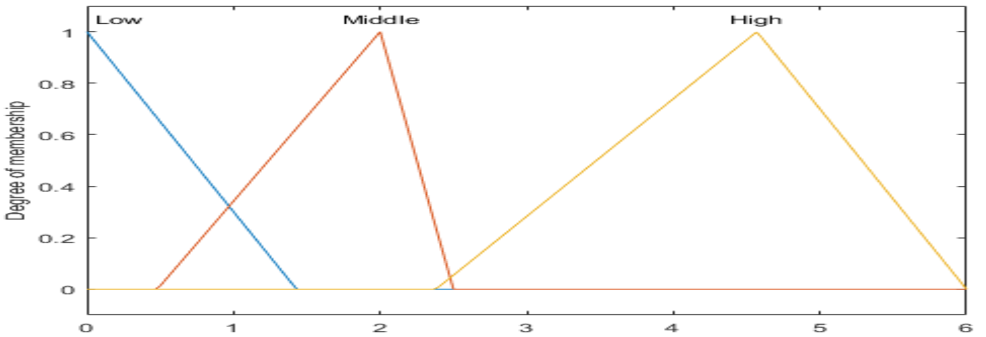
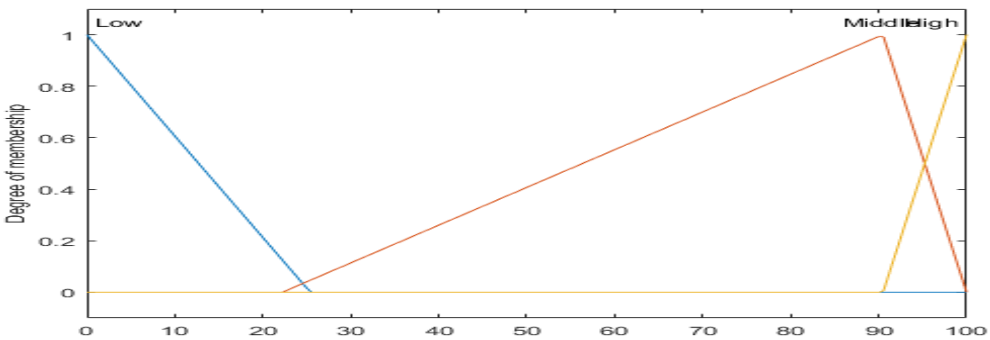


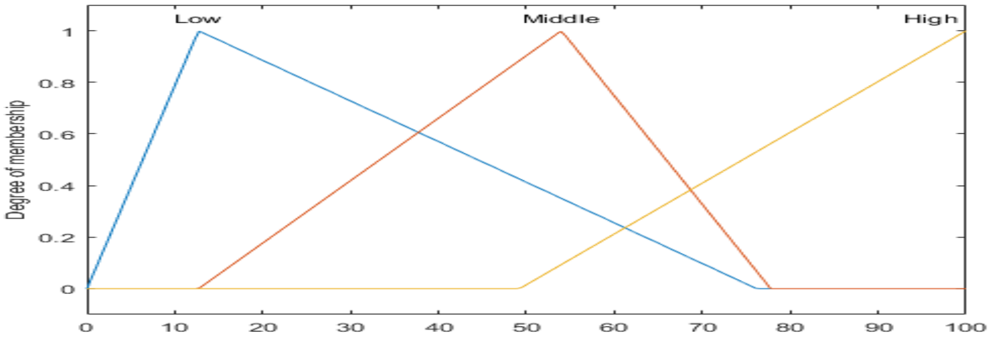
c)



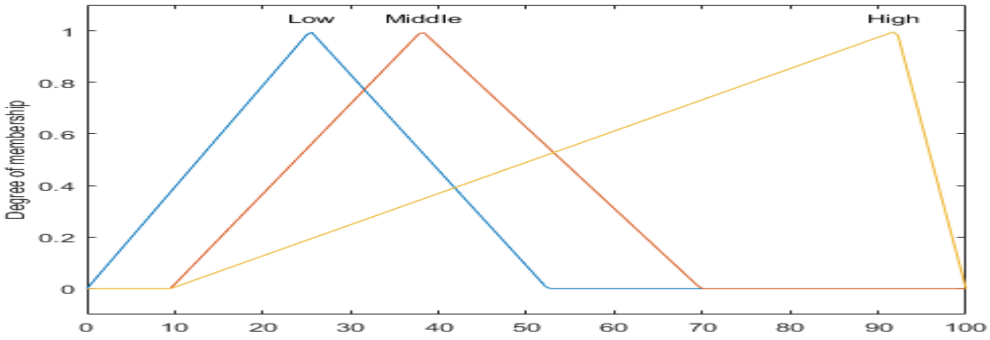
d)

Figure A3.10: Optimised membership functions for Suitability of Telecommuting for 2 days a) Drives b) Constraints/Facilitators c) Preference d) Suitability of Telecommuting for 2 days

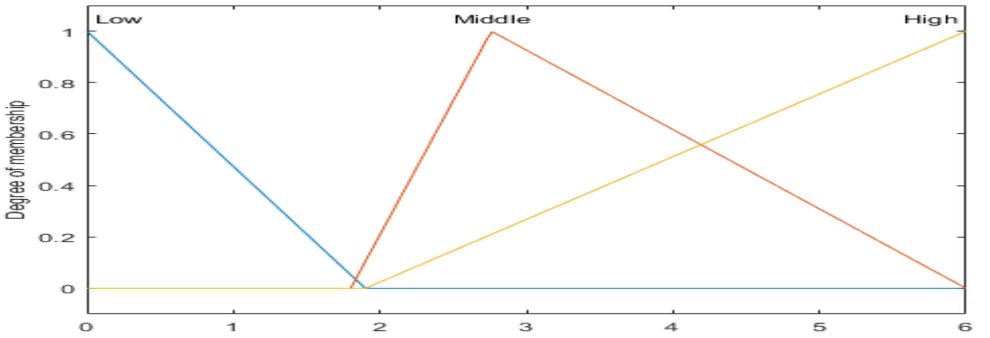
k) Suitability of for 3–days telecommuting related variables



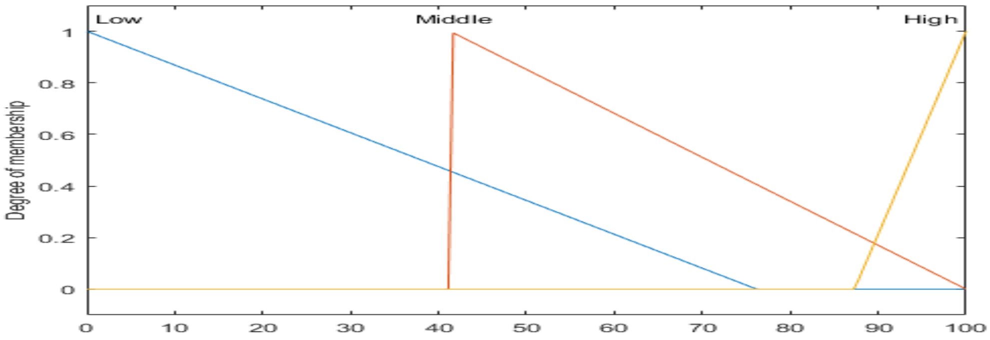
a)



b)



c)



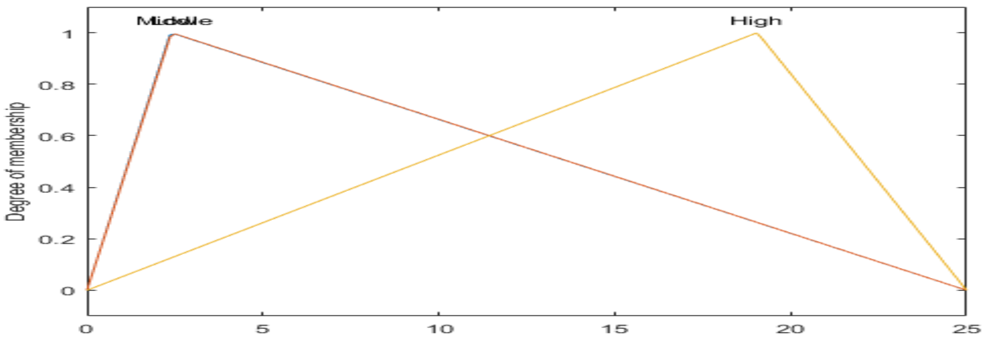
d)

Figure A3.11: Optimised membership functions for Suitability of Telecommuting for 3 days a) Drives b) Constraints/Facilitators c) Preference d) Suitability of Telecommuting for 3 days

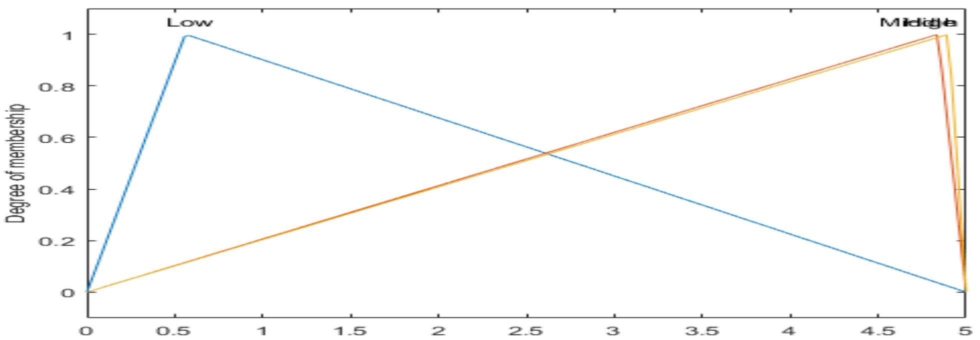
Appendix 4

Optimised membership functions for modelling suitability of telecommuting using Standard Fuzzy System:

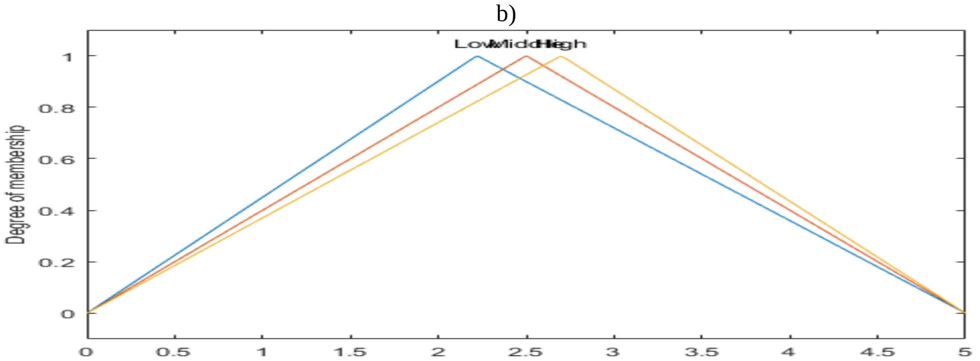
a) Optimised membership functions for modelling suitability of telecommuting 0 day:



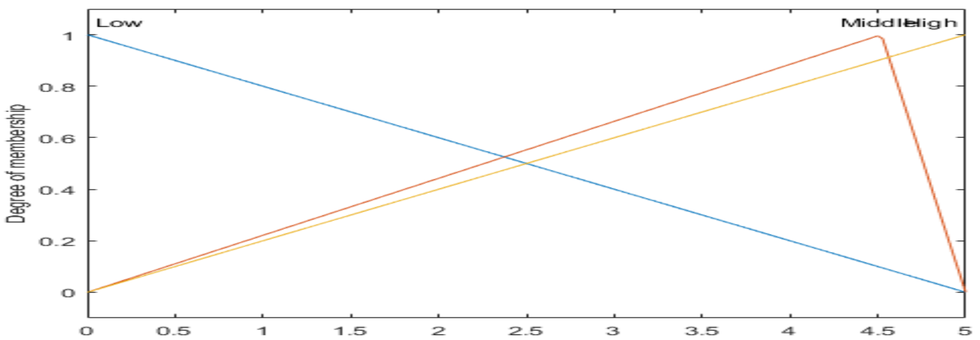
a)



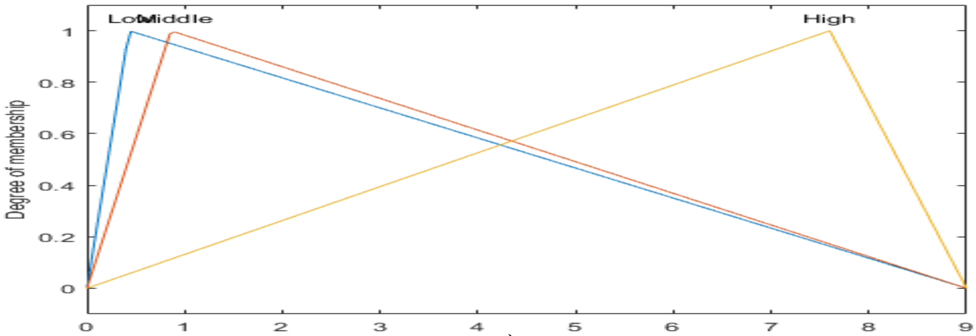
b)



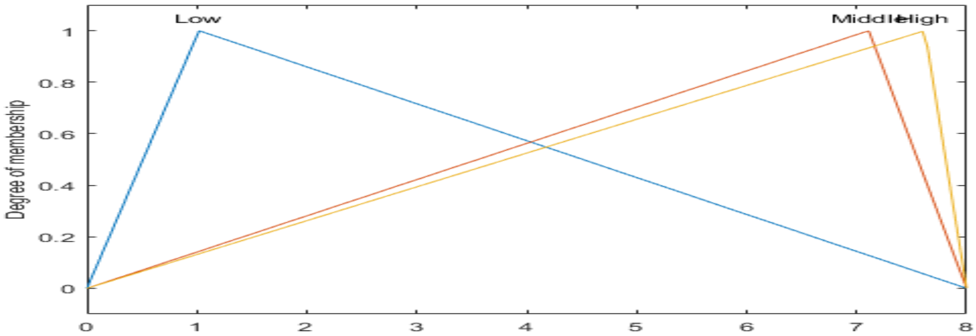
c)



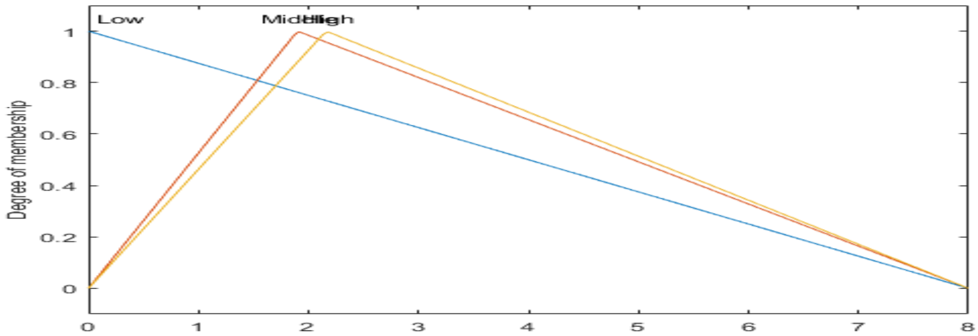
d)


Middle
High
Degree of membership

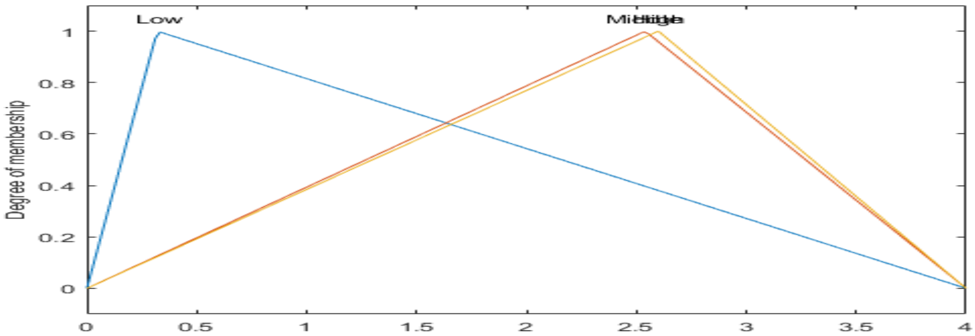
e)


Low
Degree of membership

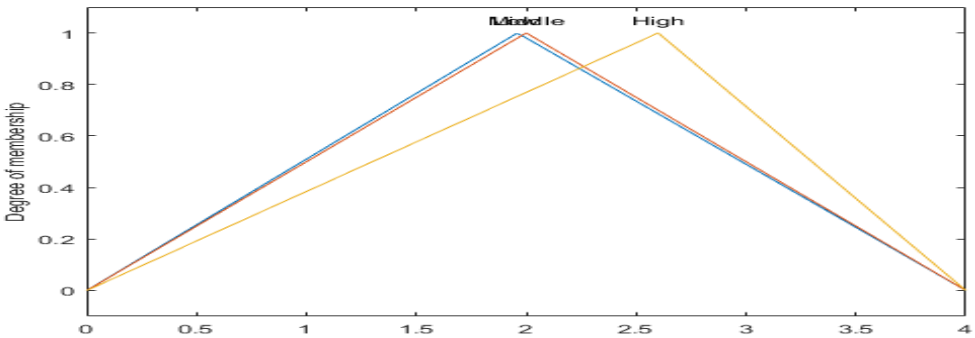
f)


Low
Degree of membership

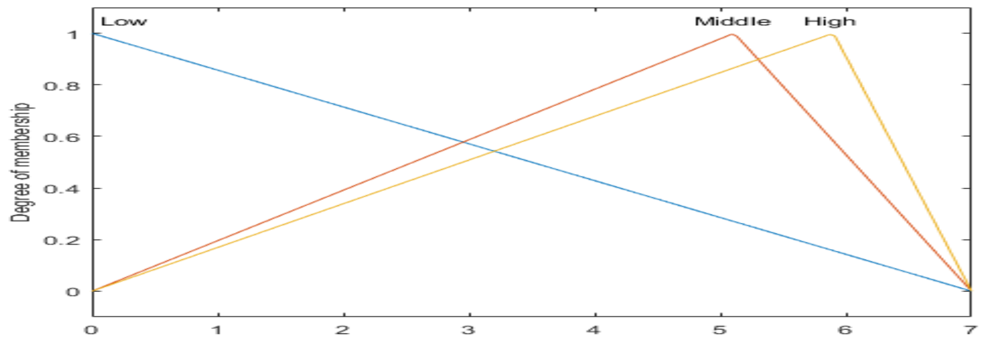
g)


Low
Degree of membership

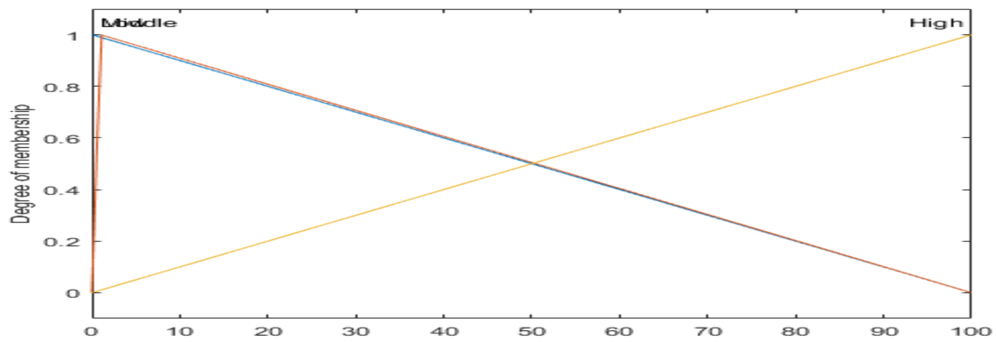
h)


High
Degree of membership

i)



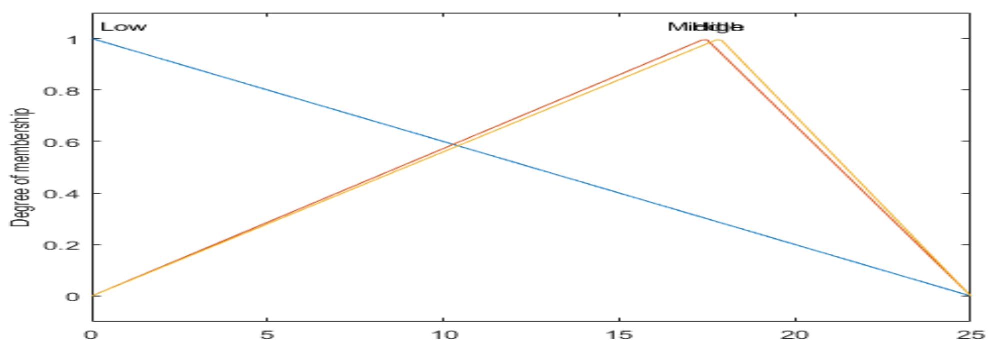
j)



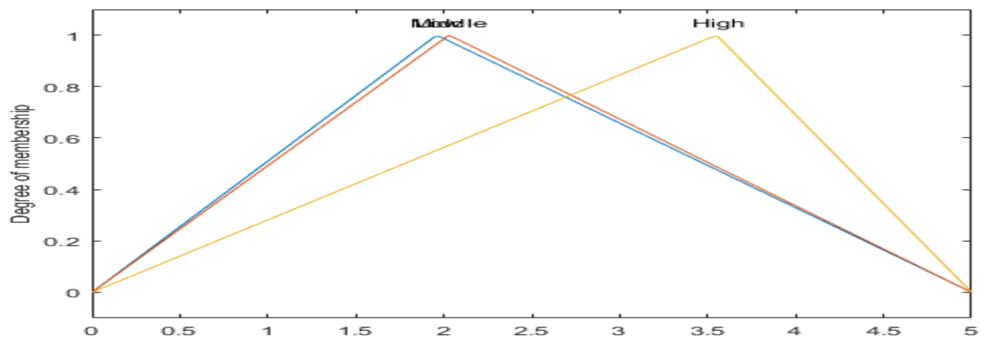
k)

Figure A4.1: Optimised membership functions for variables in Suitability of Telecommuting for 0 day
a) Work experience (current job) b) Education c) Ideology in productivity in job d) Marital Status e) Job category
f) Time-spent on PC g) Time-spent on Phone/Fax machine h) Importance of special places i) Importance of PC j) Preferences
k) Output

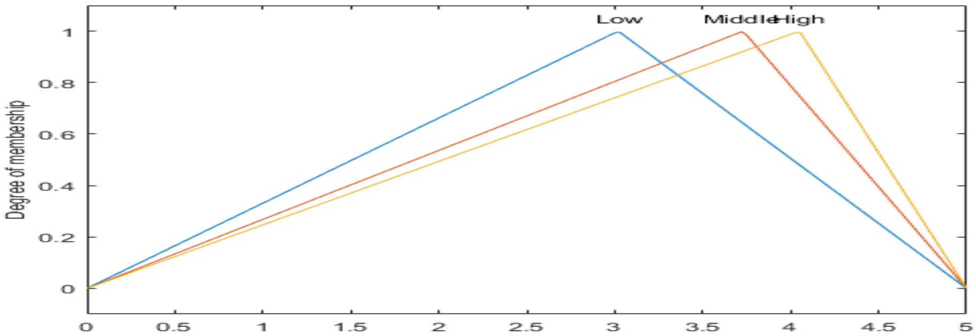
b) Optimised membership functions for modelling suitability of telecommuting 1 day:



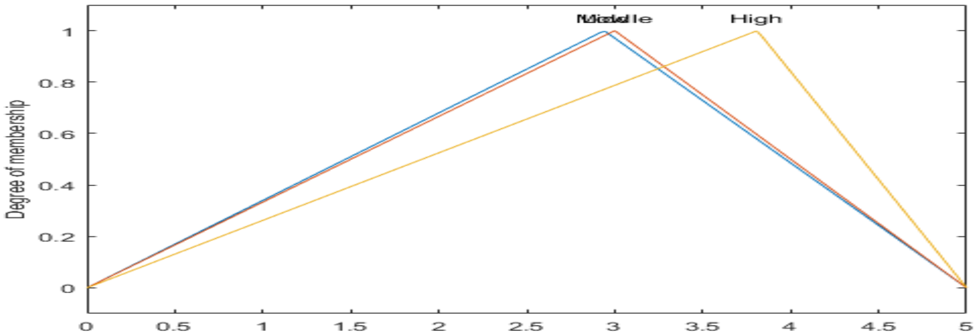
a)



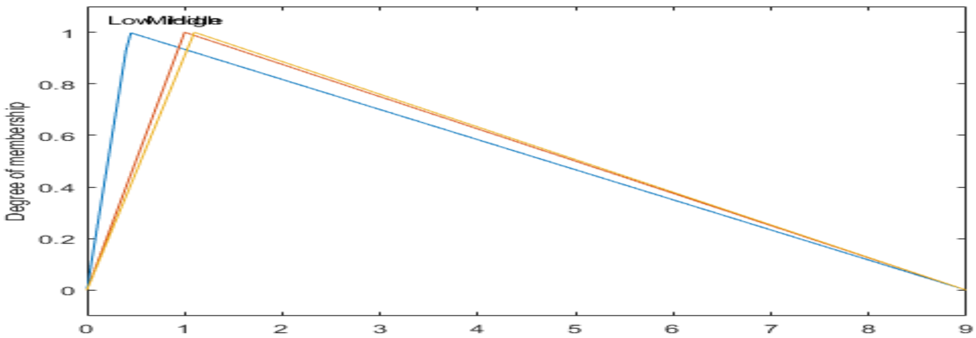
b)


Low
Middle
High
Degree of membership

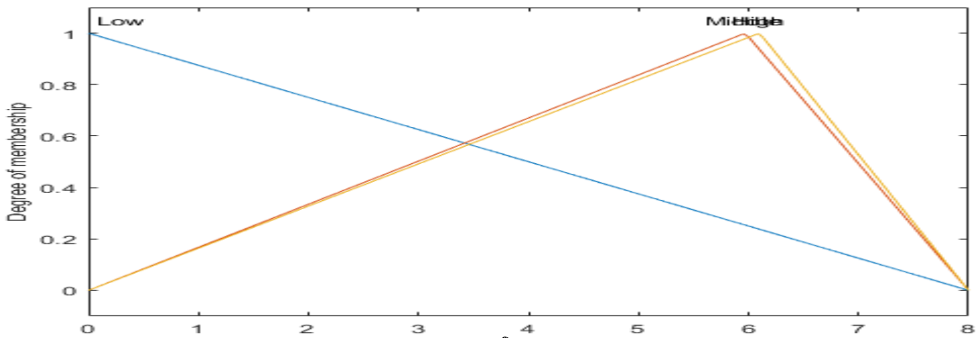
c)


High
Degree of membership

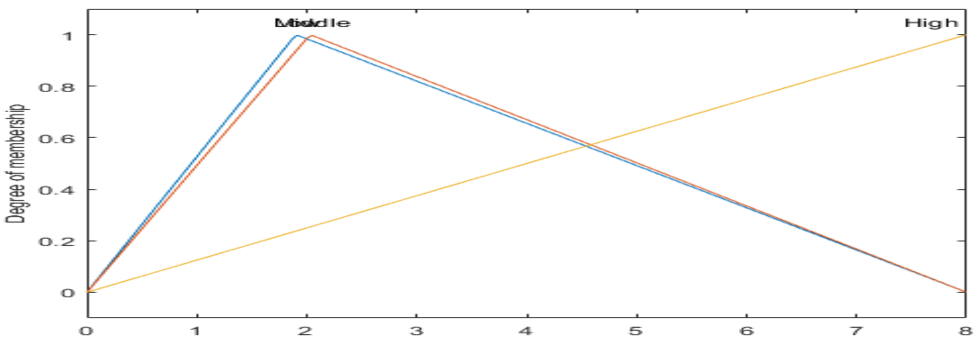
d)


Low
Degree of membership

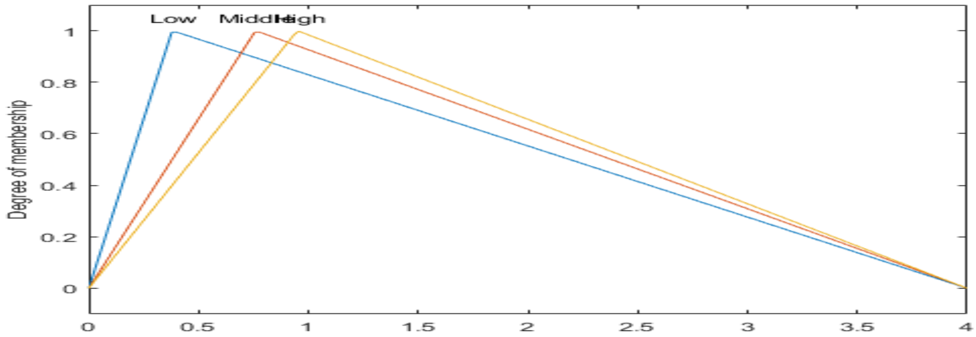
e)


Low
Degree of membership

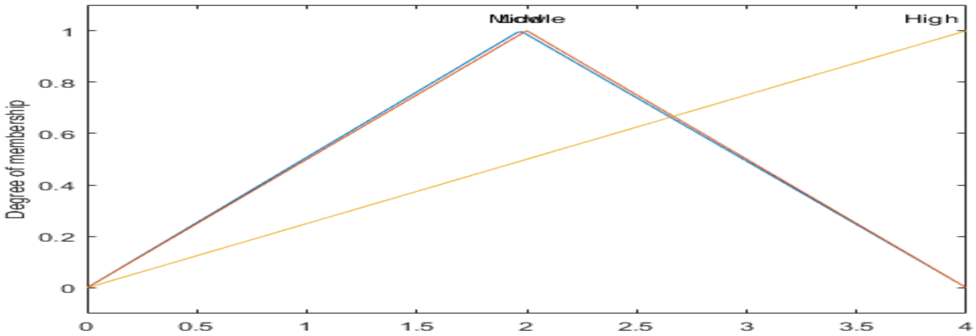
f)


High
Degree of membership

g)



h)



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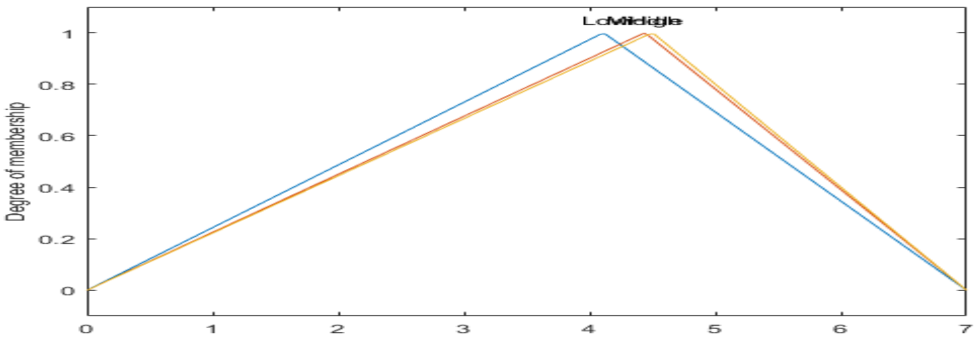

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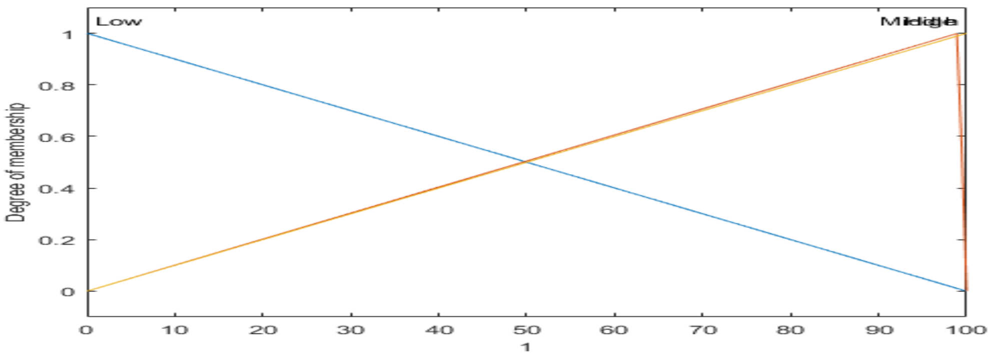
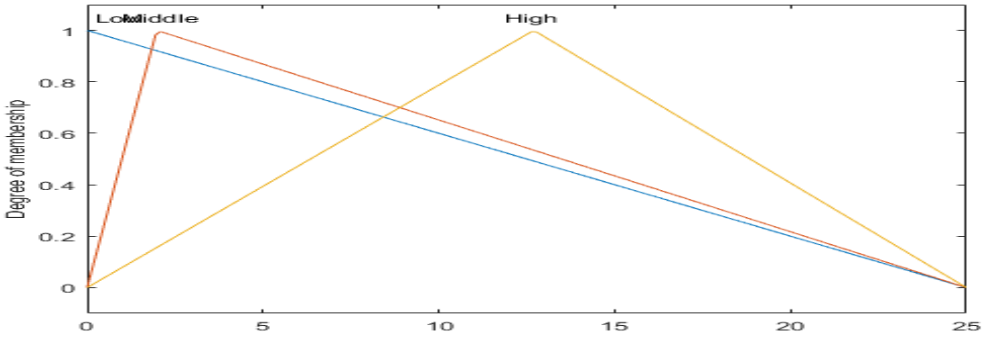


k)

Figure A4.2: Optimised membership functions for variables in Suitability of Telecommuting for 1 day
a) Work experience (current job) b) Education c) Ideology in productivity in job d) Marital Status e) Job category
f) Time-spent on PC g) Time-spent on Phone/Fax machine h) Importance of special places i) Importance of PC j) Preferences
k) Output

c) Optimised membership functions for modelling suitability of telecommuting 1 day:



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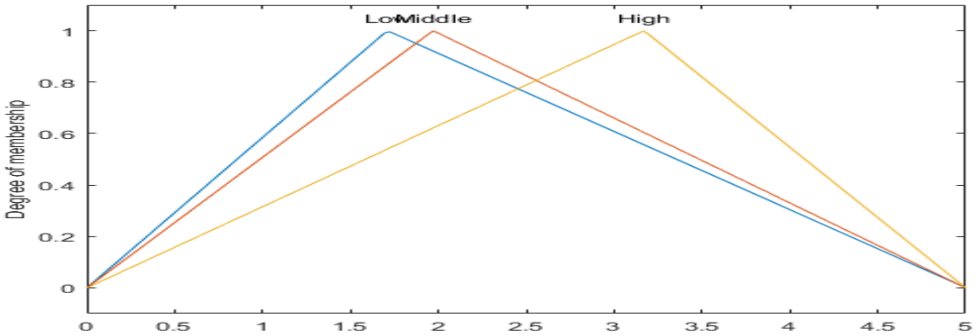
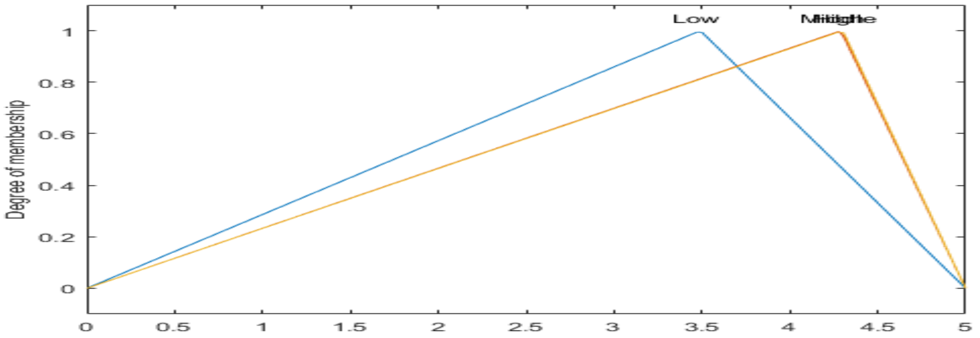
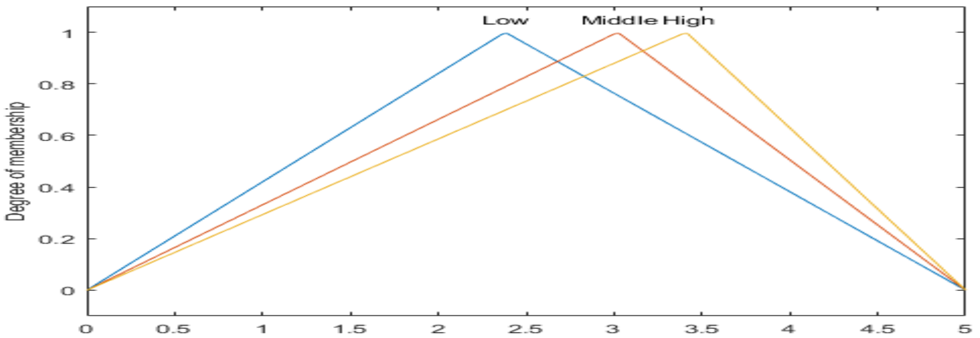


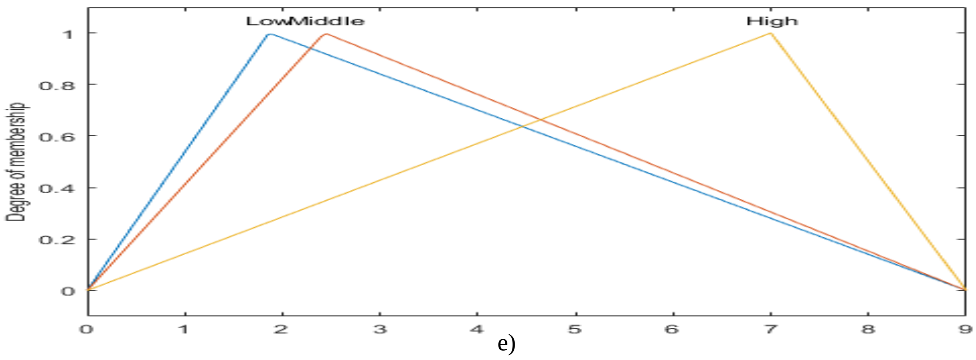
b)



c)

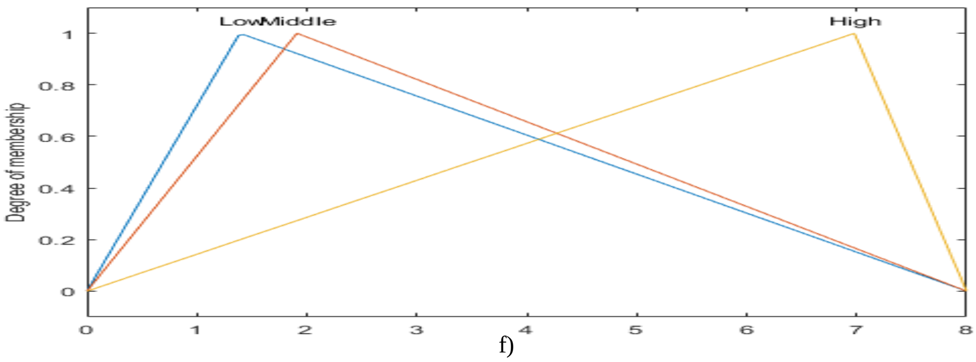


d)



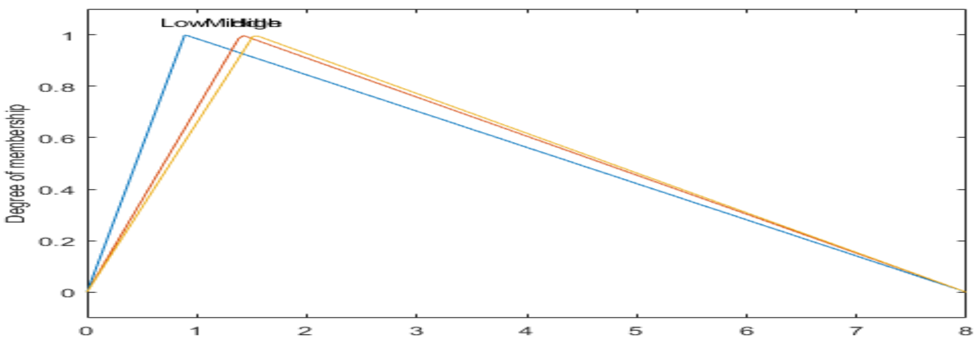
Low
Middle
High
Degree of membership

e)



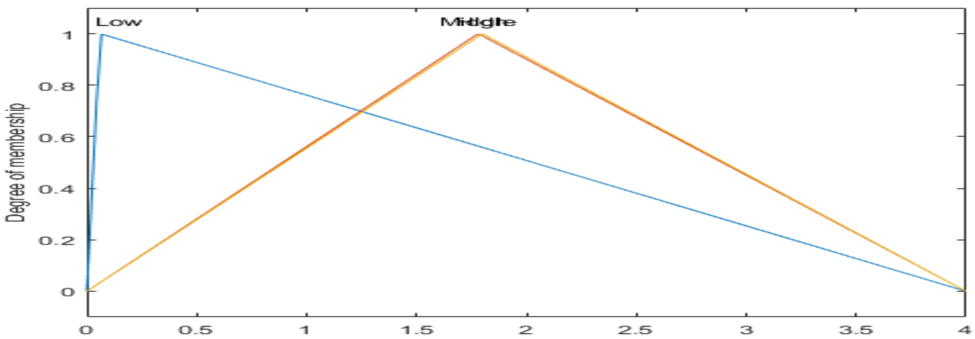
Low
Middle
High
Degree of membership

f)



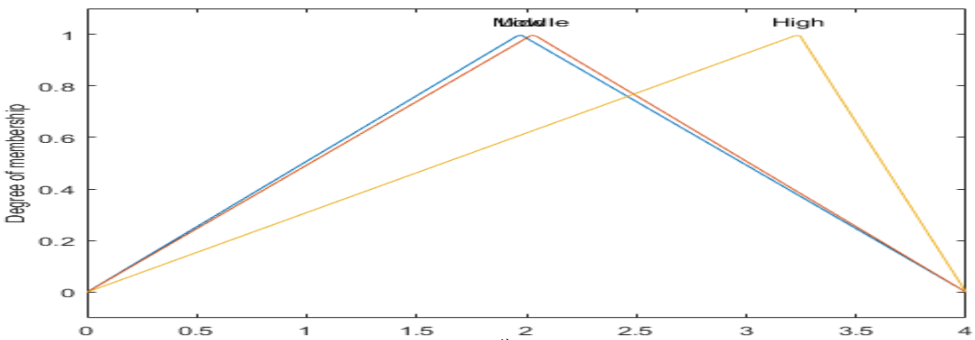
Degree of membership

g)



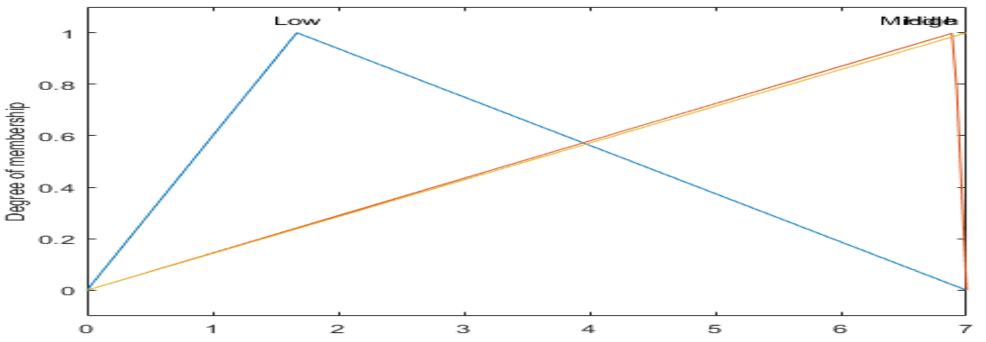
Low
Degree of membership

h)

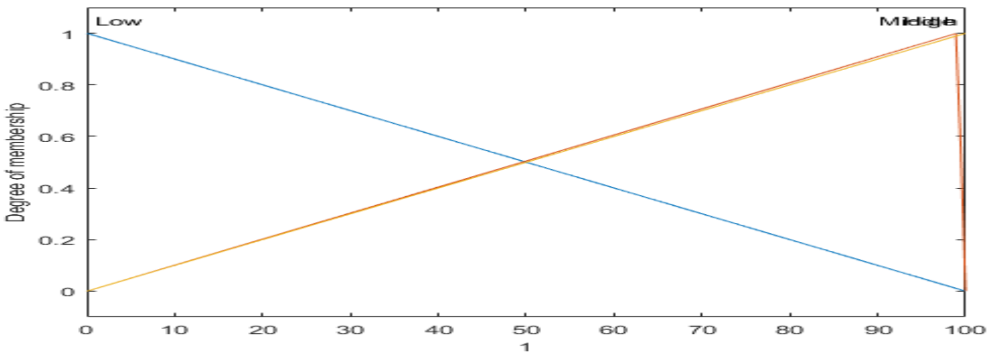


High
Degree of membership

i)



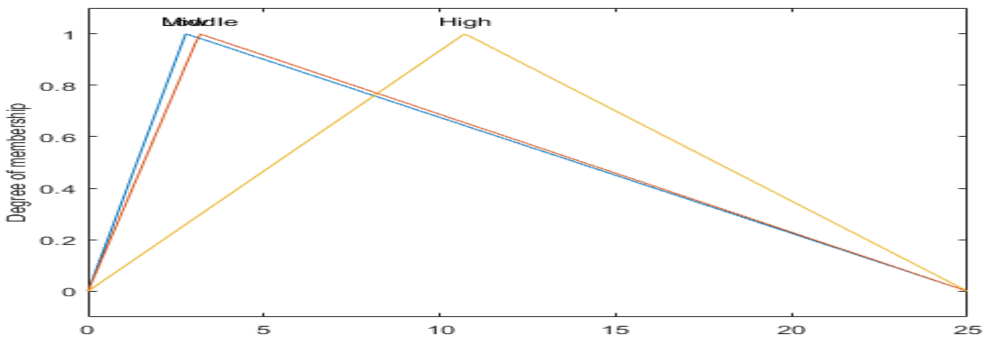
j)



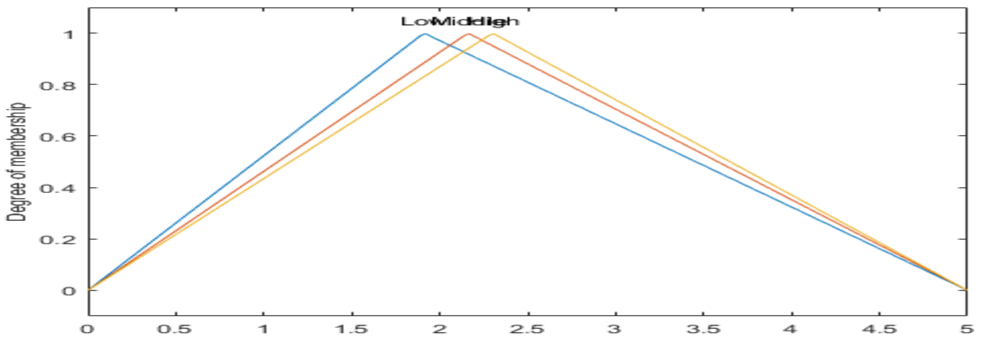
k)

Figure A4.3: Optimised membership functions for variables in Suitability of Telecommuting for 2 days
a) Work experience (current job) b) Education c) Ideology in productivity in job d) Marital Status e) Job category
f) Time-spent on PC g) Time-spent on Phone/Fax machine h) Importance of special places i) Importance of PC j) Preferences
k) Output

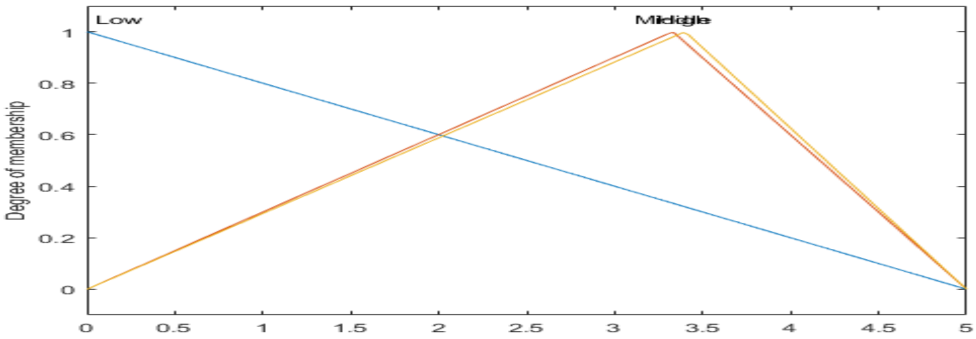
d) Optimised membership functions for modelling suitability of telecommuting 3 days:



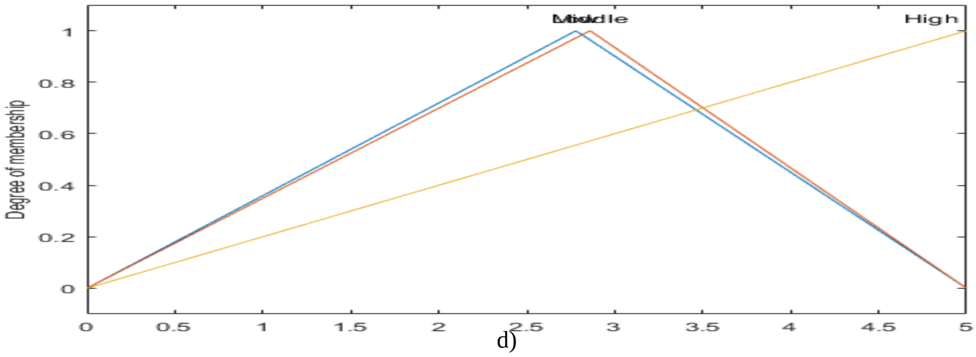
a)



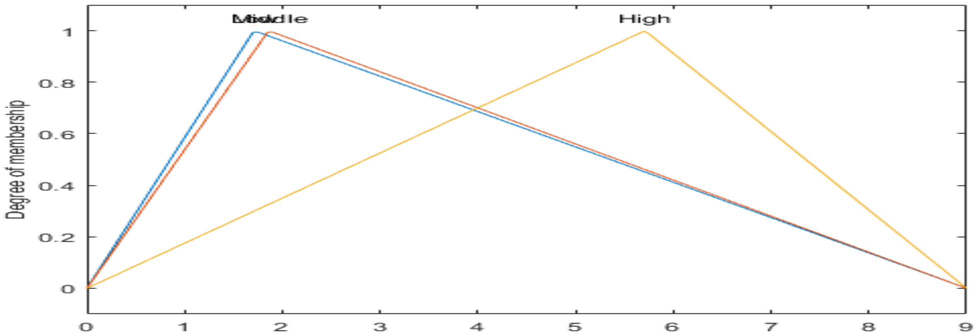
b)


Low
Degree of membership

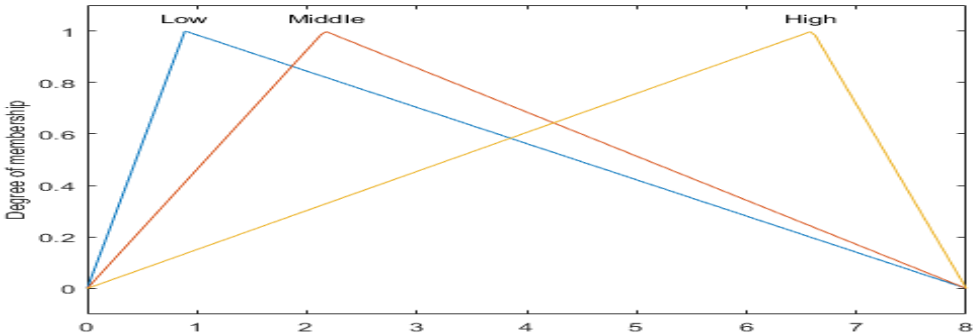
c)


High
Degree of membership

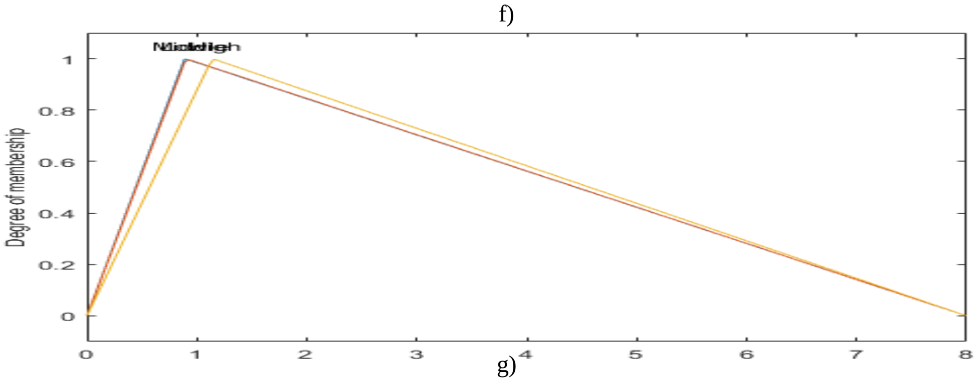
d)


High
Degree of membership

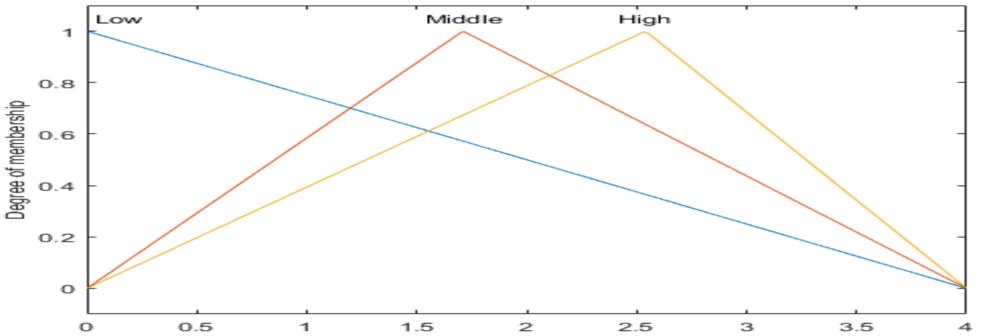
e)


Low
Middle
High
Degree of membership

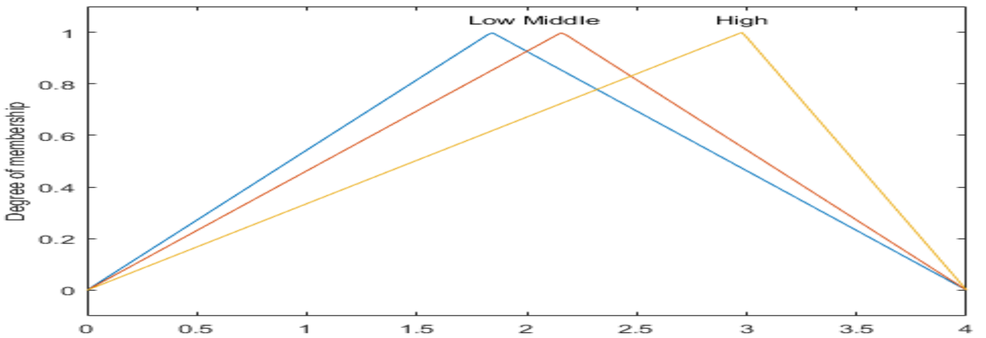
f)


Degree of membership

g)



h)



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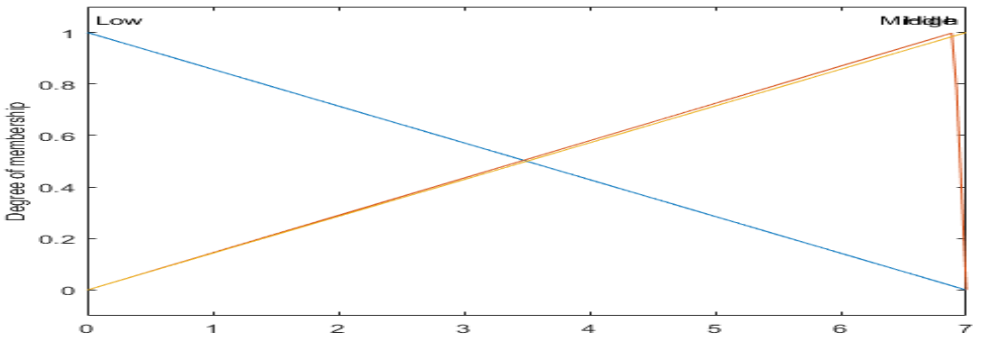
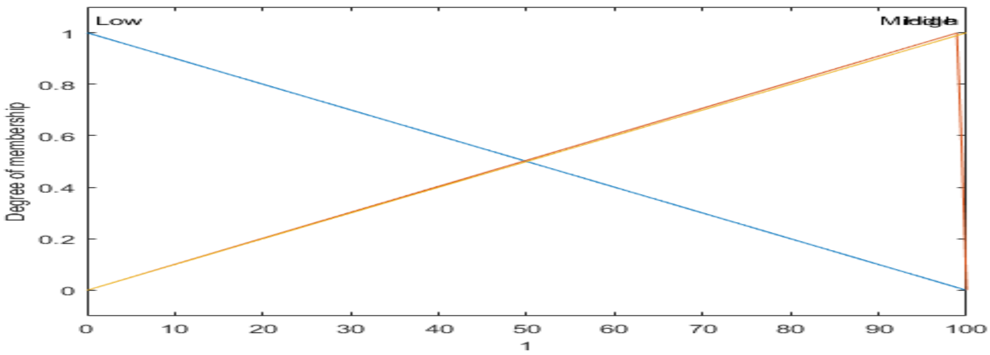


j)



k)

Figure A4.4: Optimised membership functions for variables in Suitability of Telecommuting for 3 days
a) Work experience (current job) b) Education c) Ideology in productivity in job d) Marital Status e) Job category
f) Time-spent on PC g) Time-spent on Phone/Fax machine h) Importance of special places i) Importance of PC j) Preferences
k) Output

FORM UPR16

Research Ethics Review Checklist

Please include this completed form as an appendix to your thesis (see the Postgraduate Research Student Handbook for more information

Postgraduate Research Student (PGRS) Information			**Student ID:**	638926
PGRS Name:	Farzad Arabikhan			
Department:	School of Computing	**First Supervisor:**	Dr Alexander Gegov	
Start Date: (or progression date for Prof Doc students)		**1/6/2012**		

Study Mode and Route:	Part-time ☐	MPhil ☐	MD ☐
	Full-time ☒	PhD ☒	Professional Doctorate ☐

Title of Thesis:	Telecommuting Choice Modelling Using Fuzzy Rule Based Networks
Thesis Word Count: (excluding ancillary data)	43120

If you are unsure about any of the following, please contact the local representative on your Faculty Ethics Committee for advice. Please note that it is your responsibility to follow the University's Ethics Policy and any relevant University, academic or professional guidelines in the conduct of your study

Although the Ethics Committee may have given your study a favourable opinion, the final responsibility for the ethical conduct of this work lies with the researcher(s).

UKRIO Finished Research Checklist:
(If you would like to know more about the checklist, please see your Faculty or Departmental Ethics Committee rep or see the online version of the full checklist at: http://www.ukrio.org/what-we-do/code-of-practice-for-research/)

a) Have all of your research and findings been reported accurately, honestly and within a reasonable time frame?	YES ☒ NO ☐
b) Have all contributions to knowledge been acknowledged?	YES ☒ NO ☐
c) Have you complied with all agreements relating to intellectual property, publication and authorship?	YES ☒ NO ☐
d) Has your research data been retained in a secure and accessible form and will it remain so for the required duration?	YES ☒ NO ☐
e) Does your research comply with all legal, ethical, and contractual requirements?	YES ☒ NO ☐

Candidate Statement:

I have considered the ethical dimensions of the above named research project, and have successfully obtained the necessary ethical approval(s)

Ethical review number(s) from Faculty Ethics Committee (or from NRES/SCREC):	06E6-0284-BD8B-627E-3943-5CC8-2B

If you have *not* submitted your work for ethical review, and/or you have answered 'No' to one or more of questions a) to e), please explain below why this is so:

N/A

Signed *(PGRS)*:		**Date:** 30/11/2016